

5. [10 points] Compute the **exact value** of each of the following. You do not need to show work.

a. [2 points] Find the radius of convergence  $R$  of  $\sum_{n=1}^{\infty} \frac{5(x-1)^n}{3^n}$ .

*Solution:* By the geometric series test this series converges if

$$\frac{|x-1|}{3} < 1$$

which implies that the radius of convergence is  $R = 3$ .

**Answer:**  $R =$  3

b. [2 points]  $\sum_{n=1}^{100} e^n$

**Answer:**  $\sum_{n=1}^{100} e^n =$   $\frac{e^{101}-e}{e-1}$

c. [2 points]  $\lim_{n \rightarrow \infty} \int_{-\infty}^n p(t) dt$ , where  $p(t)$  is a probability density function.

**Answer:**  $\lim_{n \rightarrow \infty} \int_{-\infty}^n p(t) dt =$  1

d. [2 points] Find the function  $f(x)$  satisfying  $\int x^3 e^x dx = x^3 e^x - \int f(x) dx$

*Solution:* Taking the derivative of both sides we find that

$$x^3 e^x = 3x^2 e^x + x^3 e^x - f(x).$$

Therefore  $f(x) = 3x^2 e^x$ .

**Answer:**  $f(x) =$   $3x^2 e^x$

e. [2 points]  $\sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!}$

*Solution:* After rewriting the series as

$$\sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{(2n)!},$$

we recognize this as  $\cos(2)$ .

**Answer:**  $\sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!} =$   $\cos(2)$