

7. [6 points] The function $r(t)$, defined for all real numbers t , gives the position of a particle moving along the unit circle,

$$r(t) = (\cos(t - t^3), \sin(t - t^3)).$$

- a. [3 points] Find all values of t where the particle stops moving.

Solution: The particle stops moving when its speed is zero. The speed is given by

$$\sqrt{(-\sin(t - t^3)(1 - 3t^2))^2 + (\cos(t - t^3)(1 - 3t^2))^2} = |1 - 3t^2|.$$

Therefore the speed is zero at $t = \pm \frac{1}{\sqrt{3}}$.

Answer: $t = \underline{\hspace{10em} \pm \frac{1}{\sqrt{3}} \hspace{10em}}$

- b. [3 points] For which values of t is the particle moving counterclockwise?

Solution: The parametric function $r(t)$ moves counterclockwise precisely when $f(t) = t - t^3$ is increasing, which is the same as $f'(t) > 0$. Since $f'(t) = 1 - 3t^2$, this happens for t in $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

Answer: $\underline{\hspace{10em} -\frac{1}{\sqrt{3}} < t < \frac{1}{\sqrt{3}} \hspace{10em}}$