9. [7 points] Brontel Muskell claims his phone's screen was broken by a penny falling from the top of the New Toledo television tower and now he wants money from the insurance company. Giuseppe Li, the actuary in charge of the case, does not believe Brontel's claim.
The differential equation modelling the velocity v of a falling object subject to air resistance is

$$v' = g - rv^2 \qquad (1)$$

where v is downward velocity, in m/s, g is acceleration due to gravity, and r is a positive constant depending on the shape and size of the falling object.

a. [3 points] The positive equilibrium solution $v = v_T$ to (1) is called the *terminal velocity*. Find v_T in terms of g and r. Is v_T stable or unstable?

Solution: Solving $0 = g - rv^2$ for v we find $v = \sqrt{\frac{g}{r}}$. This is a stable equilibrium.

Answer: $v_T =$ $\sqrt{\frac{g}{r}}$

v_T is (circle one) $\boxed{\text{STABLE}}$ UNSTABLE

b. [4 points] Giuseppe speaks with the insurance company's lead scientist, Tammy Toppel, who conducts some experiments and concludes that a penny needs to be moving at least 30 m/s to break the phone screen.

Use the company's estimates that $r = \frac{1}{40}$ for a falling penny and $g = 10$ to compute the value of v_T. Then use this value and your answer about the stability of v_T to help Dr. Toppel write a response to Brontel's claim.

Answer: Terminal velocity: 20 **m/s**

This finding (circle one) SUPPORTS $\boxed{\text{CONTRADICTS}}$

Brontel's claim because... *(briefly give reasoning below)*

Solution: Since $v_T = 20$ meters per second is a stable equilibrium one would expect a falling penny from a great height to be moving at very nearly this speed. Since this is slower than the speed required for a penny to break Brontel's phone screen, it seems he's lying.