

10. [8 points] The following problems are unrelated.

a. [3 points] Which of the following are solutions to the differential equation  $y' = x + y$ ? Circle all correct answers.

i.  $y = -1 - x + 3e^x$

v.  $y = e^x + x$

ii.  $y = 1 - x + 9e^x$

vi.  $y = e^{x^2/2}$

iii.  $y = -2 - x + e^x$

vii. NONE OF THESE

iv.  $y = -1 - x + 7e^x$

b. [3 points] Suppose  $\sum_{n=0}^{\infty} a_n(x-2)^n$  is a power series with interval of convergence  $(-1, 5]$ . Which of the following statements **must** be true? Circle all that are correct.

i.  $\sum_{n=0}^{\infty} 3^n a_n$  converges conditionally.

iv.  $\sum_{n=0}^{\infty} a_n$  converges conditionally.

ii.  $\sum_{n=0}^{\infty} 3^n a_n$  converges absolutely.

v.  $\sum_{n=0}^{\infty} a_n$  converges absolutely.

iii.  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 3$

vi.  $\sum_{n=1}^{\infty} \frac{|a_n|}{n}$  diverges.

vii. NONE OF THESE

c. [2 points] For what value of  $\beta$  does  $\int_{\pi/18}^{\beta} \sqrt{\sin^2(3\theta) + 9 \cos^2(3\theta)} d\theta$  give the length of the arc along the polar curve  $r = \sin(3\theta)$  in the first quadrant and outside the circle  $r = 1/2$ ? Circle the **one** best answer.

i.  $-\pi/18$

ii.  $\pi/18 + 2\pi$

iii.  $\pi - \pi/18$

iv.  $\pi/2 - \pi/18$

v.  $\pi/3 - \pi/18$

vi. NONE OF THESE