

6. [11 points] Consider the function $g(x)$ defined for all real numbers represented by the Taylor series

$$g(x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^{2n-1}}{(n+1)!} x^{2n}.$$

- a. [3 points] Find the values of $g^{(2019)}(0)$ and $g^{(2020)}(0)$. You do *not* need to simplify.

Answer: $g^{(2019)}(0) =$ _____ $g^{(2020)}(0) =$ _____

- b. [2 points] Find $P_4(x)$, the Taylor polynomial of $g(x)$ of degree 4 near $x = 0$.

Answer: $P_4(x) =$ _____

- c. [3 points] Define

$$G(x) = \int_{-1}^x g(t) dt.$$

Use $P_4(x)$ to estimate $G(2)$.

Answer: $G(2) \approx$ _____

- d. [3 points] Use an appropriate Taylor polynomial to compute the limit

$$\lim_{x \rightarrow 0^+} \frac{g'(x)}{x}$$

Show your work carefully.

Answer: _____