7. [13 points] The parts of this problem are unrelated.
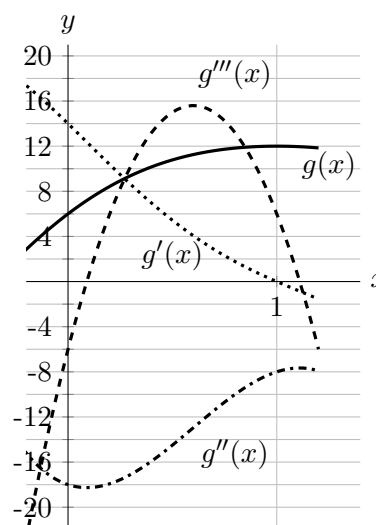
   a. [3 points] Consider the function

$$f(x) = \begin{cases} 0 & \text{for } x = 0 \\ \dfrac{\sin x}{x} - \cos x & \text{for } x \neq 0 \end{cases}$$

Find the Taylor series for $f(x)$ centered at $x = 0$. Write your answer as a single sum using sigma notation.

Answer: $f(x) = $ _____

   b. [4 points] Part of the graphs of $g(x), g'(x), g''(x)$, and $g'''(x)$ are given to the right.
   Find the third-degree Taylor polynomial for $g(x)$ near $x = 1$.

Answer: _____

   c. [6 points] Find the exact value (in closed form) of the following series. You do not need to justify your answers.

   i. $0.1 + \dfrac{0.01}{2} + \dfrac{0.001}{3} + \dfrac{0.0001}{4} + \cdots = $ _____

   ii. $\dfrac{\pi}{2} - \dfrac{3}{\pi} + \dfrac{18}{\pi^3} - \dfrac{108}{\pi^5} + \cdots = $ _____

   iii. $\dfrac{1}{2} - 2e^2 + \dfrac{2^3 e^4}{3!} - \dfrac{2^5 e^6}{5!} + \cdots = $ _____