1. [14 points] Hannah Haire and Ryan Rabbitt meet for one last race. Once again, they both start at the west side of a large square field that is 10 km wide; it will end when one reaches the east side. The racers' \((x, y)\) positions are given by the parametric equations below, where \((0, 0)\) represents the southwest corner of the field, \(x\) represents kilometers east of this corner, \(y\) represents kilometers north of this corner, and \(t \geq 0\) is measured in hours after the race begins.

Hannah Haire: \[
\begin{align*}
x &= t^2 \\
y &= \frac{t^2}{2} + 2
\end{align*}
\]

Ryan Rabbitt: \[
\begin{align*}
x &= 4t - t^2 \\
y &= t^2 - t + 1
\end{align*}
\]

Be sure to justify your answers to the following questions algebraically.

a. [2 points] Who is going faster two hours into the race?

Hannah:
\[
\text{Speed} = \sqrt{x'(t)^2 + y'(t)^2} = \sqrt{4t^2 + 2} = \sqrt{20}
\]

Ryan:
\[
\text{Speed} = \sqrt{x'(t)^2 + y'(t)^2} = \sqrt{0^2 + 3^2} = 3
\]

\text{Answer: Hannah}

b. [3 points] The race ends when the first racer reaches the east side of the field. When does the race end? Who wins?

Hannah finishes when:
\[t^2 = 10 \Rightarrow t = \sqrt{10}\]

Ryan finishes when:
\[4t - t^2 = 10 \Rightarrow t^2 - 4t + 10 = 0 \Rightarrow t = \frac{4 \pm \sqrt{16 - 40}}{2}\]

No real solutions, so Ryan never finishes.

\text{Answer: Race ends at } t = \sqrt{10} \quad \text{Winner: Hannah} \quad \text{Ryan} \quad \text{Tie}

c. [3 points] Write an integral representing the distance, in km, that Ryan runs during the race.

\[
\int_0^{\sqrt{10}} \sqrt{(4-2t)^2 + (2t-1)^2} \, dt
\]

\text{Answer:}

\[
\int_0^{\sqrt{10}} \sqrt{(4-2t)^2 + (2t-1)^2} \, dt
\]

d. [3 points] Find all times at which Ryan and Hannah are in the same spot on the field. If there are none, write "none".

\[
X \text{-values match when } t^2 = 4t - t^2 \Rightarrow 0 = 4t - 2t^2 = 2t(2-t) \Rightarrow t = 0 \text{ or } t = 2.
\]

But \(y\)-values match at neither \(t = 0\) nor \(t = 2\).

\text{Answer: } t = \text{NONE}

e. [3 points] Find all times at which Ryan is facing directly northeast (that is, halfway between directly north and directly east). If there are none, write "none".

\[
\text{NE } \Rightarrow \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dt} = \frac{dx}{dt} > 0.
\]

So \(4-2t = 2 \Rightarrow 5 = 4t \Rightarrow t = 1.25\)

At that time,
\[
\frac{dx}{dt} = 4 - 2.5 = 1.5, \quad \frac{dy}{dt} = 2.5 - 1 = 1.5, \quad \text{so both are positive.}
\]

\text{Answer: } t = 1.25