1. [14 points] Hannah Haire and Ryan Rabbit meet for one last race. Once again, they both start at the west side of a large square field that is 10 km wide; it will end when one reaches the east side. The racers' (x, y) positions are given by the parametric equations below, where $(0,0)$ represents the southwest corner of the field, x represents kilometers east of this corner, y represents kilometers north of this corner, and $t \geq 0$ is measured in hours after the race begins.

Hannah Haire: $\begin{cases} x = t^2 \\ y = \dfrac{t^2}{2} + 2 \end{cases}$ $x'(t) = 2t$
   $y'(t) = t$

Ryan Rabbitt: $\begin{cases} x = 4t - t^2 \\ y = t^2 - t + 1 \end{cases}$ $x'(t) = 4 - 2t$
   $y'(t) = 2t - 1$

Be sure to justify your answers to the following questions algebraically.

a. [2 points] Who is going faster two hours into the race?

Hannah:
Speed $= \sqrt{x'(2)^2 + y'(2)^2} = \sqrt{4^2 + 2^2} = \sqrt{20}$

Ryan:
Speed $= \sqrt{x'(2)^2 + y'(2)^2} = \sqrt{0^2 + 3^2} = 3$

Answer: _____Hannah_____

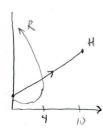b. [3 points] The race ends when the first racer reaches the east side of the field. When does the race end? Who wins?

Hannah finishes when:
$t^2 = 10 \Rightarrow t = \sqrt{10}$

Ryan finishes when:
$4t - t^2 = 10 \Rightarrow t^2 - 4t + 10 = 0 \Rightarrow t = \dfrac{4 \pm \sqrt{16 - 40}}{2}$.
No real solutions, so Ryan never finishes.

Answer: Race ends at $t = $ _$\sqrt{10}$_   Winner: [Hannah]   Ryan   Tie

c. [3 points] Write an integral representing the distance, in km, that Ryan runs during the race.

$\displaystyle\int_0^{\sqrt{10}} (\text{Ryan's speed})\, dt$

Answer: $\displaystyle\int_0^{\sqrt{10}} \sqrt{(4 - 2t)^2 + (2t - 1)^2}\, dt$

d. [3 points] Find all times at which Ryan and Hannah are in the same spot on the field. If there are none, write "none".

x -values match when $t^2 = 4t - t^2 \Rightarrow 0 = 4t - 2t^2 = 2t(2 - t)$
$\Rightarrow t = 0$ or $t = 2$.

But y-values match at neither $t = 0$ nor $t = 2$.

Answer: $t = $ __NONE__

e. [3 points] Find all times at which Ryan is facing directly northeast (that is, halfway between directly north and directly east). If there are none, write "none".

NE $\Rightarrow \dfrac{dy}{dx} = 1 \Rightarrow \dfrac{dy}{dt} = \dfrac{dx}{dt} > 0$.

So $4 - 2t = 2t - 1 \Rightarrow 5 = 4t \Rightarrow t = 1.25$

At that time,
$\dfrac{dx}{dt} = 4 - 2.5 = 1.5$, $\dfrac{dy}{dt} = 2.5 - 1 = 1.5$, so both are positive.

Answer: $t = $ __1.25__