10. [8 points] The following problems are unrelated.

a. [3 points] Which of the following are solutions to the differential equation $y' = x + y$? Circle all correct answers.

- i. $y = -1 - x + 3e^x$ — $y' = -1 + 3e^x$
- ii. $y = 1 - x + 9e^x$ — $y' = -1 + 9e^x$
- iii. $y = -2 - x + e^x$ — $y' = -1 + e^x$
- iv. $y = -1 - x + 7e^x$ — $y' = -1 + 7e^x$
- v. $y = e^x + x$ — $y' = e^x + 1$
- vi. $y = e^{x^2/2}$ — $y' = x e^{x^2/2}$
- vii. NONE OF THESE

$F(x)$

b. [3 points] Suppose $\sum_{n=0}^{\infty} a_n (x-2)^n$ is a power series with interval of convergence $(-1, 5]$. Which of the following statements **must** be true? Circle all that are correct.

$F(5)$

- i. $\sum_{n=0}^{\infty} 3^n a_n$ converges conditionally.
- ii. $\sum_{n=0}^{\infty} 3^n a_n$ converges absolutely.
- iii. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 3$

$f(3)$

- iv. $\sum_{n=0}^{\infty} a_n$ converges conditionally.
- v. $\sum_{n=0}^{\infty} a_n$ converges absolutely.
- vi. $\sum_{n=1}^{\infty} \frac{|a_n|}{n}$ diverges.
- vii. NONE OF THESE

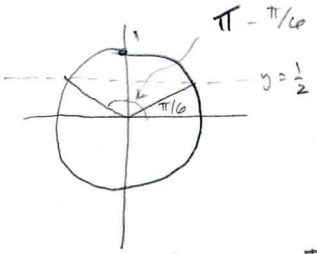
$|3^n a_n| = |(-3)^n a_n| \geq (-3)^n a_n$

$\sum (-3)^n a_n$ diverges since it's what you get when you plug in -1.

So $\sum |3^n a_n|$ diverges, $\Rightarrow \sum 3^n a_n$ converges conditionally

c. [2 points] For what value of β does $\int_{\pi/18}^{\beta} \sqrt{\sin^2(3\theta) + 9\cos^2(3\theta)}\, d\theta$ give the length of the arc along the polar curve $r = \sin(3\theta)$ in the first quadrant and outside the circle $r = 1/2$? Circle the **one** best answer.

- i. $-\pi/18$
- ii. $\pi/18 + 2\pi$
- iii. $\pi - \pi/18$
- iv. $\pi/2 - \pi/18$
- v. $\pi/3 - \pi/18$
- vi. NONE OF THESE



$\sin 3\theta \geq \frac{1}{2}$ if $\frac{\pi}{6} \leq 3\theta \leq \pi - \frac{\pi}{6}$

$\iff \frac{\pi}{18} \leq \theta \leq \frac{\pi}{3} - \frac{\pi}{18}$