10. [8 points] The following problems are unrelated.

a. [3 points] Which of the following are solutions to the differential equation $y' = x + y$? Circle all correct answers.

i. $y = -1 - x + 3e^x$
   $y' = -1 + 3e^x$

ii. $y = 1 - x + 9e^x$
   $y' = -l + 9e^x$

iii. $y = -1 - x + e^x$
   $y' = -1 + e^x$

iv. $y = -1 - x + 7e^x$
   $y' = -1 + 7e^x$

v. $y = e^x + x$
   $y' = e^x + 1$

vi. $y = e^{x/2}$
   $y' = \frac{1}{2}e^{x/2}$

vii. NONE OF THESE

b. [3 points] Suppose $\sum_{n=0}^{\infty} a_n (x - 2)^n$ is a power series with interval of convergence $(-1, 5]$.

Which of the following statements must be true? Circle all that are correct.

i. $\sum_{n=0}^{\infty} 3^n a_n$ converges conditionally.

ii. $\sum_{n=0}^{\infty} 3^n a_n$ converges absolutely.

iii. $\sum_{n=0}^{\infty} (-3)^n a_n$ diverges since $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 3$.

iv. $\sum_{n=0}^{\infty} a_n$ converges conditionally.

v. $\sum_{n=0}^{\infty} a_n$ converges absolutely.

vi. $\sum_{n=1}^{\infty} \frac{|a_n|}{n}$ diverges.

vii. NONE OF THESE

c. [2 points] For what value of $\beta$ does $\int_{-\pi/18}^{\beta} \sqrt{\sin^2(3\theta) + 9\cos^2(3\theta)} \, d\theta$ give the length of the arc along the polar curve $r = \sin(3\theta)$ in the first quadrant and outside the circle $r = 1/2$? Circle the one best answer.

i. $-\pi/18$

ii. $\pi/18 + 2\pi$

iii. $\pi - \pi/18$

iv. $\pi/2 - \pi/18$

v. $\pi/3 - \pi/18$

vi. NONE OF THESE

\[
\sin 3\theta > \frac{1}{2} \quad \text{if} \quad \frac{\pi}{18} \leq \theta \leq \frac{\pi}{3} - \frac{\pi}{18}
\]