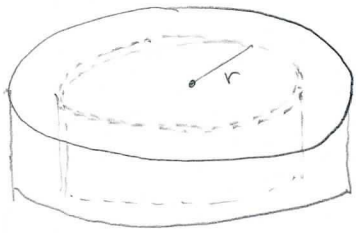
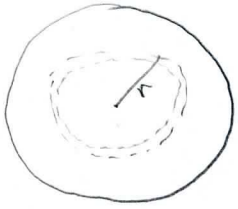
3. [10 points] A group of scientists of S.H.I.E.L.D. are investigating the Battle of Sokovia, trying to understand how Ultron lifted the capital city of Sokovia up into the sky. They use data available to them to model the situation.
Pay careful attention to the units involved in the data they use.

a. [5 points] The scientists find that they can model the part of the city that was lifted by the shape of a cylinder of radius 2 kilometers and height 100 meters. The density $\delta(r)$, in kilograms per cubic meter, is a function of distance r meters away from the central axis of the cylinder. Let M be the total mass, in kilograms, of the part of the city that was lifted. Write an expression involving one or more integrals that gives the value of M.

Density in kg/m³, so convert km to m



Top view



Shape of slice = cylindrical shell
radius " " = r $(0 \le r \le 2000\ m)$
height " " = 100 m
area " " = $(2\pi r)(\text{height}) = 200\pi r\ m^2$
thickness " " = Δr m
volume " " = $200\pi r\,\Delta r\ m^3$
mass " " = $(\text{volume})(\text{density}) = 200\pi r\,\delta(r)\,\Delta r$ kg

Answer: $M = \int_0^{2000} 200\pi r\,\delta(r)\,dr$ kg

b. [5 points] You may use M and $\delta(r)$ from part **a.** for this part.
Ultron lifted the city at a constant rate of 2 meters per second to a height of 1000 meters above the ground. While he lifted it, a small portion of the city kept detaching from the rising part at a constant rate of p kilograms per second. Write an expression involving one or more integrals that gives the total work, in Joules, it takes to complete the lifting process. Your answer may be in terms of $m, g, \delta(r)$, and M, where g is the gravitational constant, $g \approx 9.8$ m/s².

Total Time to lift = $1000\ m / 2\ m/s = 500$ s
mass at time t = $M - pt$ kg
weight " " " = $(M-pt)g$ N
Distance lifted from time t to $t+\Delta t$ = $2\Delta t$ m
Work done from time t to $t+\Delta t$ = $(\text{weight})(\text{distance}) = 2(M-pt)g\,\Delta t$ J

Answer: $\int_0^{500} 2(M-pt)g\,dt$ joules