5. [9 points] Determine whether each of the following series converges or diverges. Fully justify your answer, including carefully showing all work for any computations. Include any convergence tests used.

a. [4 points] \[ \sum_{n=1}^{\infty} \frac{3 - \sin(n^4)}{n^2} \]

Justification:

\[-1 \leq \sin(n^4) \leq 1 \]

So \[ 2 \leq 3 - \sin(n^4) \leq 4 \]

So \[ \frac{2}{n^2} \leq \frac{3 - \sin(n^4)}{n^2} \leq \frac{4}{n^2} \]

\[ \sum_{n=1}^{\infty} \frac{4}{n^2} \text{ converges by the p-test } (p=2) \]

So since \[ \frac{3 - \sin(n^4)}{n^2} \text{ is positive, } \sum \frac{3 - \sin(n^4)}{n^2} \text{ converges by comparison.} \]

b. [5 points] \[ \sum_{n=2}^{\infty} \frac{1}{n \sqrt{n}} \]

Justification:

Integral test:

\[ \int_{2}^{\infty} \frac{1}{x \sqrt{x}} \, dx = \lim_{b \to \infty} \int_{2}^{b} \frac{1}{x \sqrt{x}} \, dx \]

Let \( w = \ln x \)

\[ dw = \frac{1}{x} \, dx \]

\( x = 2 \implies w = \ln 2 \)

\( x = b \implies w = \ln b \)

\[ \int_{\ln 2}^{\infty} \frac{1}{w} \, dw = \int_{\ln 2}^{\infty} \frac{1}{w^2} \, dw \]

which diverges by the p-test \( (p=\frac{1}{2}) \).

So since \[ \frac{1}{n \sqrt{n}} \text{ is positive and decreasing, } \sum \frac{1}{n \sqrt{n}} \text{ diverges by the integral test.} \]