5. [9 points] Determine whether each of the following series converges or diverges. Fully justify your answer, including carefully showing all work for any computations. Include any convergence tests used.

a. [4 points] $\sum_{n=1}^{\infty} \frac{3-\sin(n^4)}{n^2}$

Circle one: $\boxed{\textbf{Converges}}$ **Diverges**

Justification:

$-1 \le \sin(n^4) \le 1$

So $2 \le 3 - \sin(n^4) \le 4$

so $\frac{2}{n^2} \le \frac{3-\sin(n^4)}{n^2} \le \frac{4}{n^2}$

$\sum_{n=1}^{\infty} \frac{4}{n^2}$ converges by the p-test $(p=2)$

So since $\frac{3-\sin(n^4)}{n^2}$ is positive, $\sum \frac{3-\sin(n^4)}{n^2}$ converges by comparison.

b. [5 points] $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Circle one: **Converges** $\boxed{\textbf{Diverges}}$

Justification:

Integral test:

$$\int_2^{\infty} \frac{1}{x\sqrt{\ln x}}\,dx = \lim_{b\to\infty} \int_2^{b} \frac{1}{x\sqrt{\ln x}}\,dx$$

let $w = \ln x$

$dw = \frac{1}{x}dx$

$x = 2 \Rightarrow w = \ln 2$

$x = b \Rightarrow w = \ln b$

$$= \lim_{b\to\infty} \int_{\ln 2}^{\ln b} \frac{1}{\sqrt{w}}\,dw = \int_{\ln 2}^{\infty} \frac{1}{w^{1/2}}\,dw$$

which diverges by the p-test $(p=\frac{1}{2})$.

So since $\frac{1}{n\sqrt{\ln n}}$ is positive and decreasing,

$\sum_{2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ diverges by the integral test.