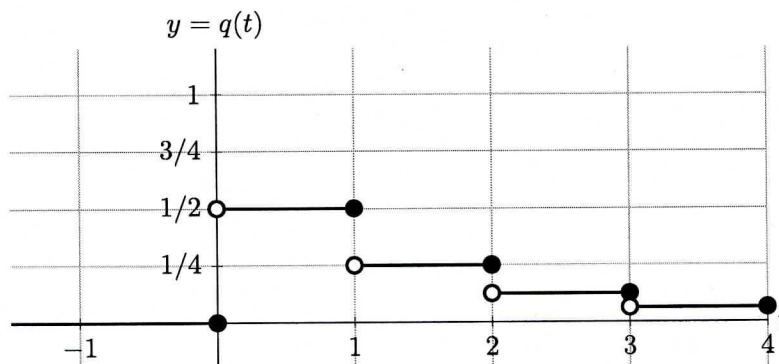


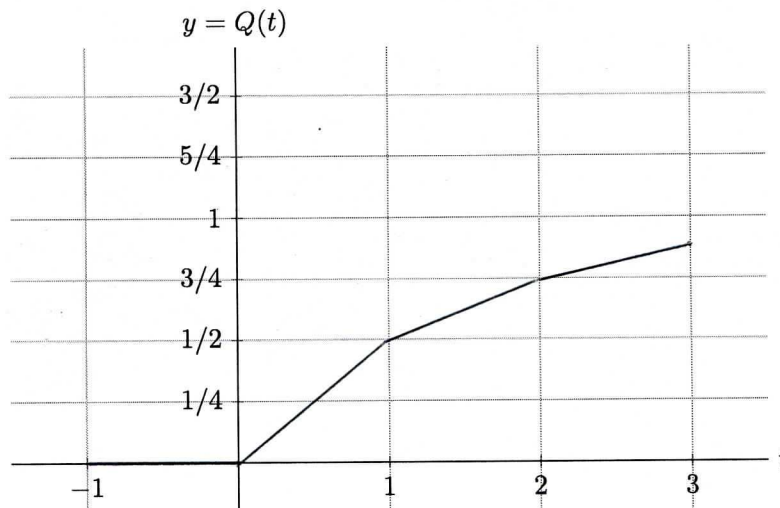
9. [7 points] After the first ever picture of a black hole was released by Event Horizon Telescope (EHT), the public awaits an image of Sgr A*, the black hole at the Galactic Center. Let t be the amount of time, in years, between now and when the EHT releases such an image. The probability density function for t is given by

$$q(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1/2^n & \text{for } n-1 < t \leq n, \text{ for each positive integer } n. \end{cases}$$

Part of the graph of $q(t)$ is given below.



- a. [4 points] Let $Q(t)$ be the cumulative distribution function for t . Carefully sketch the graph of $Q(t)$ on the domain $-1 \leq t \leq 3$.



- b. [3 points] Let P_n be the probability that EHT releases an image of Sgr A* within n years, and p_n be the probability that release time is in the n th year. For each part below, circle "True" if the statement **must be true** and circle "False" otherwise. No justification is necessary.

$$P_n = \int_{n-1}^n q(t) dt = \frac{1}{2^n}$$

The sequence p_n converges.

$$\frac{1}{2^n} \rightarrow 0$$

TRUE

FALSE

The sequence P_n converges to 0.

$$\sum_{n=1}^{\infty} p_n = \frac{1/2}{1-1/2} = 1$$

TRUE

FALSE

The series $\sum_{n=1}^{\infty} p_n$ converges.

(converges to 1)

TRUE

FALSE