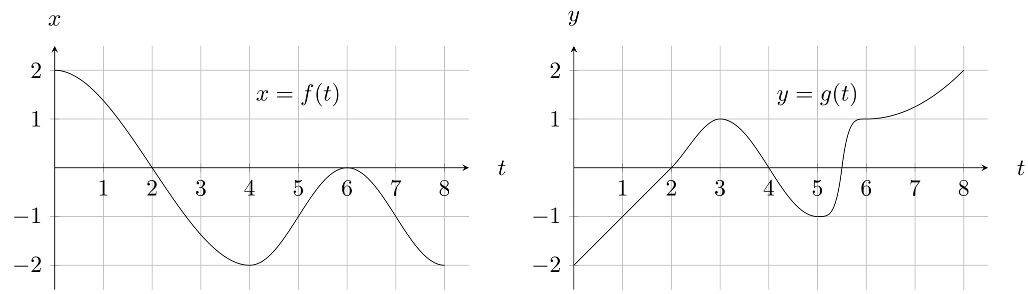
2. [13 points] To scare intruders off the island, Flora chases the intruders around. Her position at t minutes after she begins chasing the intruders is given by a parametric curve $(x, y) = (f(t), g(t))$. The graphs of $f(t)$ and $g(t)$ are given below, with x, y in km. For this question, "north" is the positive y-direction, and "east" is the positive x-direction.



a. [1 point] What is Flora's position at $t = 0$?

Solution: $f(0) = 2$ and $g(0) = -2$, so Flora is at $(2, -2)$ at $t = 0$.

b. [2 points] For $0 \le t \le 8$, at which t-value(s) is Flora at (0,0)? If there is no such time, write "NONE".

Solution: $f(t) = 0$ at $t = 2, 6$, and $g(t) = 0$ at $t = 2, 4, 5.5$, so Flora is at $(0, 0)$ only at $t = 2$.

c. [2 points] For $0 \le t \le 8$, at which t-value(s) is Flora going directly west (i.e. not in any northwest or southwest direction)? If there is no such time, write "NONE".

Solution: "Going west" means $f'(t) < 0$, and "going directly west" means Flora isn't travelling at any y-direction, i.e. $g'(t) = 0$. We have that $g'(t) = 0$ at $t = 3, 5, 6$. Among these 3 t-values, only at $t = 3$ is $f'(t) < 0$. So Flora is going directly west only at $t = 3$.

d. [2 points] For $0 \le t \le 8$, during which t-**interval(s)** is Flora going south? This includes any southeast and southwest directions, not only directly south. If there is no such time, write "NONE".

Solution: "Going south" means $g'(t) < 0$, so t needs to be in $(3, 5)$.

e. [2 points] For $0 \le t \le 8$, at which t-value(s) does Flora come to a stop? If there is no such time, write "NONE".

Solution: "Coming to a stop" means both $f'(t)$ and $g'(t)$ are 0. From the reasoning in (c), $g'(t) = 0$ when $t = 3, 5, 6$. Among these 3 points, only at $t = 6$ is $f'(t) = 0$. So Flora only comes to a stop at $t = 6$.

f. [4 points] Given that $f(1) = 4/3$, $f'(1) = -5/4$, and $g(t)$ is linear for $0 < t < 2$, find an equation for the tangent line to Flora's path at $t = 1$, given in Cartesian coordinates.

Solution: From the graph, $g(1) = -1$ and $g'(1) = 1$. The slope of the tangent line at $t = 1$ is

$$\frac{g'(1)}{f'(1)} = \frac{1}{-5/4} = -\frac{4}{5}.$$

By point-slope form, the tangent line at $t = 1$ is

$$y - (-1) = -\frac{4}{5}\left(x - \frac{4}{3}\right).$$