3. [9 points] Nile is also chasing the intruders, and her position t minutes after she starts chasing them is given by the following parametric curve (where units for both x and y are km).

$$x(t) = \frac{1}{t+1}, \qquad y(t) = t \cos t.$$

a. [4 points] Write, but do **not** evaluate, an integral to give the total distance travelled by Nile in the first minute after she started chasing the intruders.

Solution:
$$\int_0^1 \sqrt{\left(\frac{-1}{(t+1)^2}\right)^2 + (\cos t - t \sin t)^2} \ dt$$

b. [5 points] The intruders are scared and start to flee. Their total distance travelled (in km) is given by the integral

$$\int_1^\infty \sqrt{\frac{1}{u^2} + \frac{1}{e^u}} \ du$$

Does this improper integral converge or diverge? Fully justify your answer including using proper notation, and showing mechanics of any tests or theorems you use. Do not attempt to directly evaluate this integral.

Solution: For $u \geq 1$,
$$\sqrt{\frac{1}{u^2} + \frac{1}{e^u}} \geq \sqrt{\frac{1}{u^2}} = \frac{1}{u} > 0.$$

Since
$$\int_1^\infty \frac{1}{u} \ du$$

diverges by p-test, $p = 1$, the integral

$$\int_0^1 \sqrt{\frac{1}{u^2} + \frac{1}{e^u}} \ du$$

diverges by comparison test.