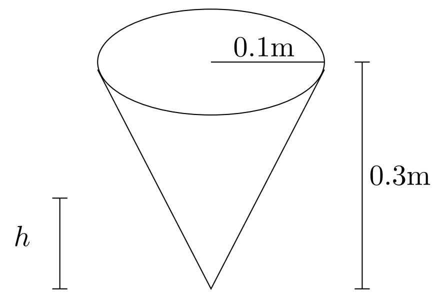
6. [8 points] Ari and Bell are enjoying their time at a beach.

a. [5 points] Ari has an ice cream cone of radius 0.1m and height 0.3m, as shown in the following picture. The cone is filled to the top with ice cream, and the ice cream located a vertical distance h meters above the bottom tip of the cone (the point at the bottom of the figure below) has density $\delta(h) = \ln(2-h)$ kg / m^3. An example of the vertical distance h is shown in the figure below.



Write, but do **not** compute, one or more integral(s) to express the total mass of the ice cream cone. Include units.

Solution: By similar triangles or setting up linear equations, radius at vertical distance h m above the bottom tip is given by

$$\frac{r}{0.1} = \frac{h}{0.3}, \qquad r = \frac{h}{3}.$$

Mass of the slice of ice cream at height h m is

$$\pi(\frac{h}{3})^2 \Delta h \ln(2-h) \text{ kg}.$$

Total mass of ice cream is

$$\int_0^{0.3} \pi(\frac{h}{3})^2 \ln(2-h)\, dh \text{ kg}.$$

b. [3 points] Bell is lifting a bottle of water straight upwards 3 meters at a constant speed. The bottle initially has a mass of 2kg, and it is leaking at a steady rate of 0.5 kg / m. Assume that gravitational acceleration is $g = 9.8$ m / s^2. Write, but do **not** compute, one or more integral(s) to express the total work done by Bell on the bottle. Include units.

Solution: Let h m be the distance of the bottle of water from its starting location at a particular instance. The mass of water at the position is $2 - 0.5h$ kg. The slice of work to lift the bottle from this position for Δh meter is given by

$$(2-0.5h)\cdot g \cdot \Delta h \text{ J}.$$

Total work is

$$\int_0^3 (2-0.5h)\cdot g\, dh \text{ J}.$$