8. [10 points] For each of the questions below, write out on your paper **all** the answers which are **always** true. No explanation is needed.

a. [3 points] Given that the power series $\sum_{n=0}^{\infty} C_n(x-1)^n$ converges at $x=3$ and diverges at $x=8$, at which of the following x-value(s) **must** the series **converge**?

-7 $\quad -6$ $\quad -3$ $\quad -1$ $\quad 0$ $\quad 2$ $\quad 6$ $\quad 9$ $\quad$ NONE OF THESE

Solution: 0,2
The power series converges at $x=3$, which is 2 away from the center $x=1$. Thus the radius of convergence is at least 2, so the power series converges at $x=0,2$, which are within 2 from the center.
The power series does not need to converge at $x=-1$ (also 2 away from the center), since $x=-1$ could potentially be an end point of the interval of convergence.

b. [3 points] Note: This part has the same set up as (a), but asks about divergence.

Given that the power series $\sum_{n=0}^{\infty} C_n(x-1)^n$ converges at $x=3$ and diverges at $x=8$, at which of the following x-value(s) **must** the series **diverge**?

-7 $\quad -6$ $\quad -3$ $\quad -1$ $\quad 0$ $\quad 2$ $\quad 6$ $\quad 9$ $\quad$ NONE OF THESE

Solution: -7,9
The power series diverges at $x=8$, which is 7 away from the center $x=1$. Thus the radius of convergence is at most 7, so the power series diverges at $x=-7,9$, which are more than 7 away from the center.
The power series does not need to converge at $x=-6$ (also 7 away from the center), since $x=-6$ could potentially be an end point of the interval of convergence.

c. [4 points] Let $x=f(t)$, $y=g(t)$ (where $0 \le t \le 10$) be a parametric curve such that $y=x^2$. Which of the following must be true?

(i) If V is the **speed** of the curve at $t=4$, then $V \ge f'(4)$.

(ii) $f'(t) \ge 0$ for $0 < t < 10$.

(iii) $g(t) \ge 0$ for $0 < t < 10$.

(iv) The tangent line to the curve at $t=1$ is $y=2x-1$.

(v) NONE OF THE ABOVE

Solution: (i), (iii)
(i): $V=\sqrt{(f'(4))^2+(g'(4))^2}$. Since $(g'(4))^2 \ge 0$, we have $V \ge \sqrt{(f'(4))^2} = |f'(4)|$.
(ii): It is possible for $f'(t) < 0$. For example, we can have $(x,y)=(-t,t^2)$.
(iii): $g(t)=y=x^2 \ge 0$.
(iv): $y=2x-1$ is the tangent line to the curve at $(1,1)$. However, the curve is not necessarily at $(1,1)$ when $t=1$. For example, if $(x,y)=(-t,t^2)$, then the curve is at $(-1,1)$ when $t=1$.