8. [10 points] For each of the questions below, write out on your paper all the answers which are always true. No explanation is needed.

   a. [3 points] Given that the power series $\sum_{n=0}^{\infty} C_n(x-1)^n$ converges at $x=3$ and diverges at $x=8$, at which of the following $x$-value(s) must the series converge?

      $-7, -6, -3, -1, 0, 2, 6, 9$ NONE OF THESE

   Solution: $0, 2$

   The power series converges at $x=3$, which is 2 away from the center $x=1$. Thus the radius of convergence is at least 2, so the power series converges at $x=0, 2$, which are within 2 from the center.

   The power series does not need to converge at $x=-1$ (also 2 away from the center), since $x=-1$ could potentially be an end point of the interval of convergence.

   b. [3 points] Note: This part has the same set up as (a), but asks about divergence.

      Given that the power series $\sum_{n=0}^{\infty} C_n(x-1)^n$ converges at $x=3$ and diverges at $x=8$, at which of the following $x$-value(s) must the series diverge?

      $-7, -6, -3, -1, 0, 2, 6, 9$ NONE OF THESE

   Solution: $-7, 9$

   The power series diverges at $x=8$, which is 7 away from the center $x=1$. Thus the radius of convergence is at most 7, so the power series diverges at $x=-7, 9$, which are more than 7 away from the center.

   The power series does not need to converge at $x=-6$ (also 7 away from the center), since $x=-6$ could potentially be an end point of the interval of convergence.

   c. [4 points] Let $x = f(t), y = g(t)$ (where $0 \leq t \leq 10$) be a parametric curve such that $y = x^2$. Which of the following must be true?

      (i) If $V$ is the speed of the curve at $t=4$, then $V \geq f'(4)$.

      (ii) $f'(t) \geq 0$ for $0 < t < 10$.

      (iii) $g(t) \geq 0$ for $0 < t < 10$.

      (iv) The tangent line to the curve at $t=1$ is $y = 2x - 1$.

      (v) NONE OF THE ABOVE

   Solution: (i), (iii)

   (i): $V = \sqrt{(f'(4))^2 + (g'(4))^2}$. Since $(g'(4))^2 \geq 0$, we have $V \geq \sqrt{(f'(4))^2} = |f'(4)|$.

   (ii): It is possible for $f'(t) < 0$. For example, we can have $(x, y) = (-t, t^2)$.

   (iii): $g(t) = y = x^2 \geq 0$.

   (iv): $y = 2x - 1$ is the tangent line to the curve at $(1, 1)$. However, the curve is not necessarily at $(1, 1)$ when $t=1$. For example, if $(x, y) = (-t, t^2)$, then the curve is at $(-1, 1)$ when $t=1$. 