9. [3 points] For $x > 0$, let $g(x)$ be a positive continuous function, and

$$G(x) = \int_x^{e^{x^2}} \frac{1}{g(t)}\, dt.$$

Find $G'(x)$. Your answer may involve g.

Solution:

$$G'(x) = \frac{1}{g(e^{x^2})} \cdot e^{x^2} \cdot 2x - \frac{1}{g(x)}.$$

10. [6 points] Compute the **radius** of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{8^n}{(n+1)^2} x^{3n+1}$$

Be sure to show all your reasoning.

Solution: Use ratio test.

$$\begin{aligned}
\lim_{n\to\infty} \left| \frac{8^{n+1}x^{3n+4}}{(n+2)^2} \cdot \frac{(n+1)^2}{8^n x^{3n+1}} \right| &= \lim_{n\to\infty} \left| \frac{8^{n+1}}{8^n} \cdot \frac{x^{3n+4}}{x^{3n+1}} \cdot \frac{(n+1)^2}{(n+2)^2} \right| \\
&= \lim_{n\to\infty} \left| 8x^3 \cdot \frac{(n+1)^2}{(n+2)^2} \right| \\
&= 8|x|^3
\end{aligned}$$

To have the power series converge, we need $8|x|^3 < 1$, i.e.

$$|x|^3 < \frac{1}{8}, \qquad |x| < \frac{1}{2}.$$

So the radius of convergence is $\frac{1}{2}$.