

1. [13 points] Universe of Movies (UofM) is a new online movie rating database, which assigns movies a rating from zero to five stars. The star ratings for each movie are not necessarily integer values. The following **probability density function** gives the distribution of star rating,  $t$ , for the films in the UofM database.

$$p(t) = \begin{cases} 0 & t \leq 0 \\ \frac{t}{6} & 0 < t \leq 3 \\ \frac{t}{32} & 3 < t \leq 5 \\ 0 & 5 < t \end{cases}$$

- a. [6 points] Write a formula for the corresponding cumulative distribution function,  $F(t)$ .

$$F(t) = \begin{cases} 0 & t \leq 0 \\ \frac{t^2}{12} & 0 < t \leq 3 \\ \frac{t^2}{64} + \frac{39}{64} & 3 < t \leq 5 \\ 1 & 5 < t \end{cases}$$

- b. [3 points] What is the median number of stars for the films in the online database? Be sure to show any work.

*Solution:* From finding the CDF in a), we know that  $\int_0^3 p(t)dt = .75$  so the median is in  $[0, 3]$ . Therefore, we just need to solve  $\int_0^M \frac{t}{6}dt = \frac{M^2}{12} = .5$  which when solved tells us that  $M = \sqrt{6}$

Answer:  $M = \sqrt{6}$

- c. [4 points] Write, but do not compute, the formula for the mean number of stars for the movies in the UofM database. Write out formulas of all functions you use, i.e. do not include  $p$  or  $F$  in your answers.

Answer:  $\int_0^3 \frac{t^2}{6}dt + \int_3^5 \frac{t^2}{32}dt$