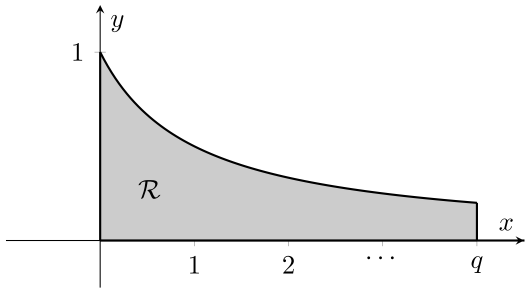
2. [11 points] Consider the function $f(x) = e^{-2x}$, and the region $\mathcal{R}$ bounded by the x-axis, the y-axis, $y = f(x)$ and $x = q$, where q is a positive constant larger than 2.



a. [4 points] Give a formula for, but do not compute, the volume of the solid formed by rotating the region $\mathcal{R}$ around the y-axis. Your answer should depend on q. *(Hint: Use the shell method)*

Solution: Using shell method, we see that $V = \int_0^q 2\pi x e^{-2x} dx$.

Using washer method, we get:

$$\int_0^{e^{-2q}} \pi q^2 dy + \int_{e^{-2q}}^1 \pi \left(\frac{1}{2}\ln(y)\right)^2 dy$$

b. [4 points] Compute the integral you found in part a). Your final answer should be in terms of q.

Solution: Using shell method from a) allows us use integration by parts with $u = x$ and $dv = e^{-2x}dx$ to give $du = dx$ and $v = \frac{-1}{2}e^{-2x}$. Putting this together gives:

$$\int_0^q 2\pi x e^{-2x} dx = -\pi x e^{-2x}\Big|_0^q + \pi \int_0^q e^{-2x} dx = \pi \left[-xe^{-2x} - \frac{1}{2}e^{-2x}\right]\Big|_0^q$$

When we evaluate FTC, we get:

$$V = \pi \left[-qe^{-2q} - \frac{1}{2}e^{-2q} - 0 + \frac{1}{2}\right]$$

c. [3 points] Taking a limit of your answer in b), compute the volume of the infinitely long solid of revolution formed by rotating the region $\mathcal{R}$ around the y-axis. Be sure to show how you got the value of your limit.

Solution: An infinitely long solid means $q \to \infty$. Therefore we get

$$V = \lim_{q\to\infty} \pi \left[qe^{-2q} - \frac{1}{2}e^{-2q} - 0 + \frac{1}{2} \right] = \pi[0 - 0 + 0 + \frac{1}{2}] = \frac{\pi}{2}.$$

where the first limit vanishes by L'Hopital or dominating functions argument, and the second is just a standard limit.