

3. [16 points] Use the table below to answer the following questions. Write your answers using **exact form** on the blank provided. If there is not enough information to complete the problem, write N.I. You need to evaluate all integrals completely, but you do not need to simplify your final answers.

$t$	0	1	2	3	4	5
$f(t)$	3	2	5	$e$	7	0
$f'(t)$	-1	-4	3	2	$\pi$	1
$f''(t)$	2	8	1	2	6	2

a. [4 points] MID(2) for  $\int_1^5 f(t)dt$

Answer: 24.

*Solution:*  $\Delta t = 2$  which means the intervals are  $[1, 3]$  and  $[3, 5]$  with midpoints then being 2 and 4 respectively.

$$f(t_1) \cdot \Delta t + f(t_2) \cdot \Delta t = f(2) \cdot 2 + f(4) \cdot 2 = 24$$

b. [4 points] TRAP (2) for  $\int_0^4 t f'(t)dt$

Answer:  $4\pi + 12$ .

*Solution:* TRAP(2) = (LEFT(2) + RIGHT(2))/2,  $\Delta t = 2$ , so we have

$$2(0f'(0) + 2 \cdot 2f'(2) + 4f'(4))/2 = 0 + 12 + 4\pi$$

c. [4 points] Average value of  $f'(2t)$  on  $[0, 2]$

Answer: 1.

*Solution:* Average value =  $\frac{\int_0^2 f'(2t)dt}{(2-0)} = \frac{1}{4}(f(4) - f(0)) = 1$

- d. [4 points] Second degree Taylor Polynomial of  $f(-x)$  around  $x = -2$ .

Answer:  $5 - 3(x + 2) + \frac{1}{2}(x + 2)^2$ .

*Solution:* Note that  $f(-x)$  at  $x = -2$  is  $f(2)$ , the first derivative is  $-f'(2)$  and the second derivative is  $f''(2)$ . Then, since we are around  $x = -2$ , we need out terms to be of the form  $(x + 2)$ . Putting this all together in the formula for second degree gives us

$$f(2) - f'(2)(x + 2) + \frac{f''(2)}{2}(x + 2)^2 = 5 - 3(x + 2) + \frac{1}{2}(x + 2)^2$$