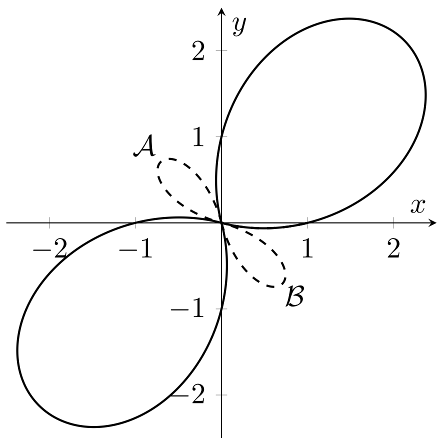
5. [16 points] The following problems relate to the polar graph shown below, defined by the polar curve $r(\theta) = 2\sin(2\theta) + 1$, on the domain $[0, 2\pi]$. Both the dashed and solid curves are part of the graph of $r(\theta)$.



a. [4 points] Find all θ values in the interval $[0, 2\pi]$ such that $r(\theta) = 0$.

Solution: We set $r(\theta) = 2\sin(2\theta) + 1 = 0$ and solve. This becomes:

$$\sin(2\theta) = \frac{-1}{2}$$

This means $2\theta = \frac{-\pi}{6} + 2k\pi$ or $\frac{7\pi}{6} + 2k\pi$ Now if we divide by 2, we get $\theta = \frac{-\pi}{12} + k\pi$ or $\frac{7\pi}{12} + k\pi$. Now, since we need to be in the interval$[0, 2]$, we take $k = 1, 2$ for the first possible term to get:

$$\theta = \frac{-\pi}{12} + \pi = \frac{11\pi}{12} \qquad \theta = \frac{-\pi}{12} + 2\pi = \frac{23\pi}{12}.$$

We take $k = 0, 1$ for the second possible term to get:

$$\theta = \frac{7\pi}{12} + 0\pi = \frac{7\pi}{12} \qquad \theta = \frac{7\pi}{12} + \pi = \frac{19\pi}{12}$$

as answers

Answers: $\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

b. [4 points] Determine the θ intervals corresponding to the dashed portions $\mathcal{A}$ and $\mathcal{B}$ of the curve above.

Solution: We need to examine when our radius is changing sign. First, we see that at $\theta = 0$, the radius is positive. It must stay positive, sweeping out the large undotted portion in the first quadrant, until $\theta = \frac{7\pi}{12}$, which is when the radius is first zero, based on our work in a). At this point, the radius becomes negative. The next zero occurs at $\frac{11\pi}{12}$, which means that for the interval $[\frac{7\pi}{12}, \frac{11\pi}{12}]$, the radius is negative, which should sweep out the dotted curve $\mathcal{B}$. Continuing in this way, we see that the next interval $[\frac{11\pi}{12}, \frac{19\pi}{12}]$ will correspond to the next solid region, and then the interval $[\frac{19\pi}{12}, \frac{23\pi}{12}]$, we trace out the curve $\mathcal{A}$.

Interval for $\mathcal{A}$: $[\frac{19\pi}{12}, \frac{23\pi}{12}]$ Interval for $\mathcal{B}$: $[\frac{7\pi}{12}, \frac{11\pi}{12}]$

c. [4 points] Write an expression involving one or more integrals for the area of the region enclosed by the **solid** curves only (do not include the region enclosed by the dashed curves).

Solution: The solid portions are where the dashed portions are not. Therefore they are defined by the angles which are not our answers in b). Since we still need to be in $[0, 2\pi]$, This means our answers are:

$$\frac{1}{2}\int_0^{\frac{7\pi}{12}} (2\sin(2\theta)+1)^2\,d\theta + \frac{1}{2}\int_{\frac{11\pi}{12}}^{\frac{19\pi}{12}} (2\sin(2\theta)+1)^2\,d\theta + \frac{1}{2}\int_{\frac{23\pi}{12}}^{2\pi} (2\sin(2\theta)+1)^2\,d\theta$$

d. [4 points] Write an expression involving one or more integrals for the total arc length of the **dashed** curves in the graph above.

Solution: We can do this two ways. The first, is to have one integral for each curve. Using the polar arclength formula we get:

$$\int_{\frac{7\pi}{12}}^{\frac{11\pi}{12}} \sqrt{(2\sin(\theta)+1)^2 + (4\cos(2\theta))^2}\,d\theta + \int_{\frac{19\pi}{12}}^{\frac{23\pi}{12}} \sqrt{(2\sin(\theta)+1)^2 + (4\cos(2\theta))^2}\,d\theta$$

The second is to double one integral. Using the polar arclength formula we get:

$$2\int_{\frac{7\pi}{12}}^{\frac{11\pi}{12}} \sqrt{(2\sin(\theta)+1)^2 + (4\cos(2\theta))^2}\,d\theta$$