7. [6 points] Compute the interval of convergence of the series:

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n4^n} (x - 4)^n \]

Show all work and give all necessary justification. You may assume it has radius of convergence 4 and you do not need to show this.

**Solution:** We have that the radius of convergence is 4. We know that the series is centered at \( x = 4 \). Therefore, to compute the interval of convergence, we just need to check convergence at the endpoints, which occur at 0 and 8. At \( x = 0 \), the series becomes:

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n4^n} (0 - 4)^n = \sum_{n=1}^{\infty} \frac{(-1)^n(-4)^n}{n4^n} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}4^n}{n4^n} = \sum_{n=1}^{\infty} \frac{1}{n}. \]

This diverges by \( p \)-test \( p = 1 \), so the series does not converge at \( x = 0 \), meaning the interval is now (0, 8?].

If we plug in \( x = 8 \), we get:

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n4^n} (8 - 4)^n = \sum_{n=1}^{\infty} \frac{(-1)^n(4)^n}{n4^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}. \]

As the terms \( a_n = \frac{1}{n} \) are decreasing and going to zero, this series converges by the alternating series test. Since the series converges at \( x = 8 \), we include it in the radius of convergence.

Interval of Convergence: \((0, 8]\)