9. [10 points] After a mistake on their last mission, Brad and Angelina must go on the run together, or risk being captured by an opposing agent. Brad and Angelina's shared position is given by the parametric equations:

$$(f(t), g(t)) = (t^2 + 10,\ 2t^2 + 10)$$

and the agent pursuing them has position given by the equations

$$(r(t), q(t)) = (7t,\ \sin(\pi t) + 12t).$$

The time t is measured in hours after Brad and Angelina have gone on the run, and all distances are given in miles.

a. [5 points] The agent catches up with Brad and Angelina at the smallest positive t-value when the agent is in the same position as they are. Find the time when the agent catches up with Brad and Angelina.

Solution: Start by comparing the x coordinates. To be in the same place at the same time, we need $t^2 + 10 = 7t$. This is the same as solving $t^2 - 7t + 10 = 0$. This polynomial factors as $(t-2)(t-5) = 0$. So the only times the x coordinates are the same is at $t = 2, 5$. Now, we just check that these work for the y-coordinates. If we plug in $t = 2$, $g(2) = 8 + 10$, where $q(2) = 0 + 24$, since $18 \neq 24$, the times $t = 2$ fails. However, if we plug in $t = 5$, we see that $g(5) = 50 + 10 = 60$ and $q(5) = 0 + 60 = 60$. So the time $t = 5$ is a solution, and therefore must be the only solution.

Answer: $t = 5$

b. [5 points] Compute the total distance traveled by Brad and Angelina before the agent catches up with them.

Solution: We will need to use the parametric arclength formula. Since the agent catches them at $t = 5$, this is the upper bound, and $t = 0$ is the lower bound. $f'(t) = 2t$, and $g'(t) = 4t$. Therefore, the arclength interal becomes:

$$\int_0^5 \sqrt{(2t)^2 + (4t^2)}dt = \int_0^5 \sqrt{4t^2 + 16t^2}dt = \int_0^5 \sqrt{20}tdt.$$

Integrating gives:

$$\int_0^5 \sqrt{20}tdt = \frac{\sqrt{20}}{2}t^2\Big|_0^5 = 25\sqrt{5}\text{miles}.$$

Answer: $25\sqrt{5}$**miles**