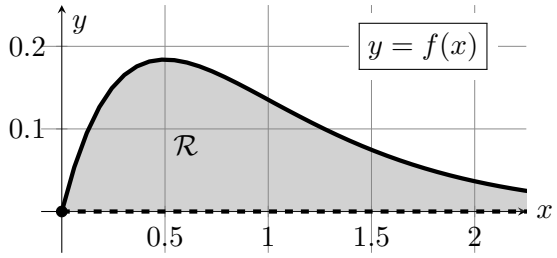
4. [10 points] Louise, the world-famous abstract artist and cheese enthusiast, has a dream about an infinitely-long cheese sculpture. The sculpture involves $\mathcal{R}$, which is the region above the x-axis and below the curve $f(x) = xe^{-2x}$ on the interval $[0, \infty)$. A portion of $\mathcal{R}$ is the shaded region below.



a. [4 points] Write an improper integral that represents the **volume** of the infinitely-long solid of revolution formed by rotating the region $\mathcal{R}$ around the x-axis.
Your answer should not involve the letter f. **Do not evaluate your integral.**

Answer: _____

b. [6 points] The **area** of the region $\mathcal{R}$ (not the volume of the rotated solid) is given by the improper integral

$$\int_0^\infty xe^{-2x}\,dx.$$

Determine whether this improper integral is convergent or divergent.
You may use either a direct computation or the comparison test to reach your conclusion.
Fully justify your answer including using **proper notation.** Circle your final answer choice.

Circle one: **Convergent** **Divergent**