1. [12 points] Let $g(x)$ be a **differentiable** function, and let $G(x)$ be a **continuous antiderivative** of $g(x)$. Some values of $g(x)$ and $G(x)$ are given in the table below:

x	-2	-1	0	1	2
$g(x)$	0	$\sqrt{3}$	4	5	-1
$G(x)$	π	$1/2$	-2	0	1

Use the table above to answer the following questions. Write your answers in **exact form**. If there is not enough information to complete a problem, write "NEI." Your answers should not involve the letters g or G, but you do not need to simplify your final answers. Show all your work.

a. [3 points] Compute the **average value** of $g'\big(g(x)\big) \cdot g'(x)$ on the interval $[-2, 2]$.

Solution: We use a substitution $u = g(x)$, so that $du = g'(x)\,dx$, and so the average value is

$$\frac{1}{2-(-2)} \int_{-2}^{2} g'\big(g(x)\big) \cdot g'(x)\,dx = \frac{1}{4} \int_{0}^{-1} g'(u)\,du = \frac{1}{4}\big(g(-1) - g(0)\big) = \frac{1}{4}(\sqrt{3} - 4).$$

Answer: $\frac{1}{4}(\sqrt{3} - 4)$

b. [3 points] Compute $F'(1)$, where $F(x) = \displaystyle\int_{x^3-2}^{4} G(t)\,dt$.

Solution: By the Second Fundamental Theorem of Calculus and the chain rule,

$$F'(x) = -3x^2 G(x^3 - 2), \qquad \text{and so} \qquad F'(1) = -3G(-1) = -3/2.$$

Answer: $-\frac{3}{2}$

c. [3 points] Approximate $\displaystyle\int_{-2}^{2} G(x)\,dx$ using TRAP(2).

Solution: The subintervals we use are $[-2, 0]$ and $[0, 2]$. The formula for TRAP(2) is thus

$$\text{TRAP}(2) = \frac{1}{2} \cdot 2\big(G(-2) + 2G(0) + G(2)\big) = \pi + 2(-2) + 1 = \pi - 3.$$

Answer: $\pi - 3$

d. [3 points] Compute $\displaystyle\lim_{x\to\infty} x\,G(1 + \tfrac{1}{x})$.

Solution: Since $\lim_{x\to\infty} G(1 + \frac{1}{x}) = G(1) = 0$, the given limit is of the indeterminate form $\infty \cdot 0$. Recalling that $G'(x) = g(x)$, we rearrange and apply L'Hôpital's rule:

$$\lim_{x\to\infty} x\,G(1 + \tfrac{1}{x}) = \lim_{x\to\infty} \frac{G(1 + \frac{1}{x})}{\frac{1}{x}} \overset{L'H\,\frac{0}{0}}{=} \lim_{x\to\infty} \frac{-\frac{1}{x^2} g(1 + \frac{1}{x})}{-\frac{1}{x^2}} = \lim_{x\to\infty} g(1 + \tfrac{1}{x}) = g(1) = 5.$$

Answer: 5