2. [12 points] In the video game *Super Maria 64*, sisters Maria and Luisa travel through the Lilypad Kingdom to collect magical Rainbow Crystals. In the Sandland Desert, represented by the xy-plane, the sisters run around collecting all the Rainbow Crystals they see.
All distances in this problem are measured in kilometers. For $t \geq 0$, the sisters' positions t hours after they start running are given by the following parametric equations:

$$\text{Maria: } \begin{cases} x(t) = t^2 + t - 6 \\ y(t) = 2\sin(\pi t) \end{cases} \qquad \text{Luisa: } \begin{cases} x(t) = 2t^2 - 4t \\ y(t) = \cos(\frac{\pi}{2}t) \end{cases}$$

a. [2 points] Find **Maria's position** 1 hour after the sisters start running.

Solution: Maria's (x, y) position at $t = 1$ is given by $(x(1), y(1))$, where for Maria, $x(1) = -4$ and $y(1) = 0$.

Answer: $x =$ -4 $y =$ 0

b. [3 points] Find **Maria's speed,** in kilometers per hour, 1 hour after the sisters start running.

Solution: Maria's instantaneous speed at $t = 1$ is given by $\sqrt{x'(1)^2 + y'(1)^2}$, where for Maria, $x'(t) = 2t + 1$ and $y'(t) = 2\pi\cos(\pi t)$, so $x'(1) = 3$ and $y'(1) = -2\pi$.

Answer: $\sqrt{3^2 + (-2\pi)^2}$

c. [3 points] Find **all** times $t \geq 0$ at which **Luisa** travels **directly north** (that is, not in any northwest or northeast direction). If there is no such time, write "NONE." Show your work to justify your answer.

Solution: In order for Luisa to travel directly north, we must have $x'(t) = 0$ and $y'(t) > 0$, where for Luisa, $x'(t) = 4t - 4$ and $y'(t) = -\frac{\pi}{2}\sin(\frac{\pi}{2}t)$. The only value of t that satisfies $x'(t) = 0$ is $t = 1$. However, $y'(1) = -\frac{\pi}{2} < 0$, so Luisa travels directly **south** at the time $t = 1$. Therefore there are no times t where Luisa travels directly north.

Answer: $t =$ **NONE**

d. [4 points] Find **all** times $t \geq 0$ at which Maria and Luisa are at the same position. If there is no such time, write "NONE." Show your work to justify your answer.

Solution: The sisters are at the same x-position when $t^2 + t - 6 = 2t^2 - 4t$. We solve for t:

$$\begin{aligned} t^2 + t - 6 &= 2t^2 - 4t \\ 0 &= t^2 - 5t + 6 \\ 0 &= (t-2)(t-3) \end{aligned}$$

So, Maria and Luisa are at the same x-position when $t = 2$ and $t = 3$. However, observe that Maria and Luisa are not at the same y-position when $t = 2$ since $2\sin(2\pi) \neq \cos(\frac{\pi}{2} \cdot 2)$. On the other hand, they are at the same y-position at $t = 3$ since $2\sin(3\pi) = \cos(\frac{\pi}{2} \cdot 3)$. So $t = 3$ is the only time at which they are at the same position.

Answer: $t =$ 3