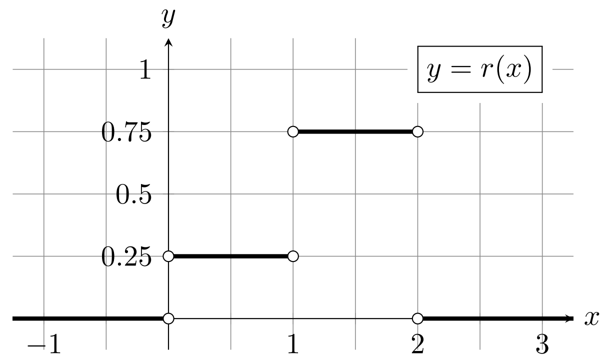
6. [10 points]
A survey has recently been conducted on the University of Michigan campus which asked a large number of students to choose a random real number in the interval $[0, 2]$.

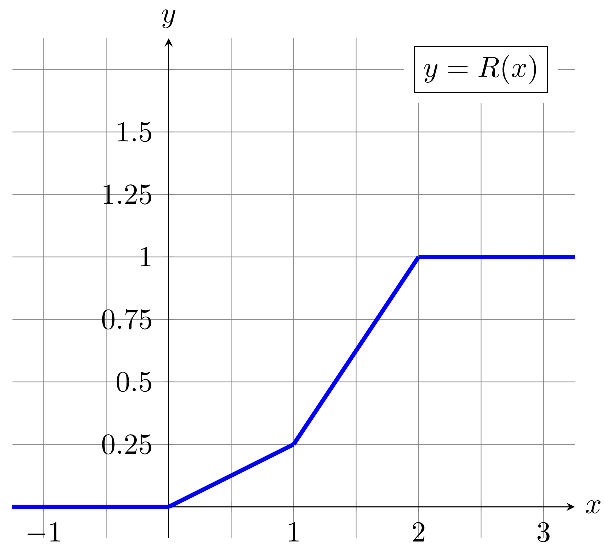
The numbers chosen by students are described by the **probability density function** (pdf) $r(x)$. A graph of $r(x)$ is shown to the right.



You do not need to show your work in this problem, but partial credit may be given for work shown.

a. [5 points]
Let $R(x)$ be the **cumulative distribution function** (cdf) corresponding to $r(x)$.
The function $R(x)$ is defined for all real numbers x.
On the axes provided to the right, sketch a graph of $R(x)$ **on the interval** $[-1, 3]$.
Be sure to pay attention to:

- where $R(x)$ is and is not differentiable;
- where $R(x)$ is increasing, decreasing, or constant;
- where $R(x)$ is concave up, concave down, or linear;
- the values of $R(x)$ at $x = -1, 0, 1, 2$, and 3.



b. [2 points] Compute the fraction of students that chose a number in the interval $[1, 2]$.

Solution: There are two ways to solve this problem:

Solution 1 (Using the pdf $r(x)$)**:** The answer is given by the quantity $\int_1^2 r(x)\,dx$. Using the graph of $r(x)$, we have $\int_1^2 r(x)\,dx = \int_1^2 0.75\,dx = 0.75$.

Solution 2 (Using the cdf $R(x)$)**:** The answer is given by the quantity $R(2) - R(1)$. Using our graph of $R(x)$ from part (a), we have $R(2) - R(1) = 1 - 0.25 = 0.75$.

Answer: 3/4 **or** 0.75 **or** 75%

c. [3 points] Compute the **median** of the numbers chosen among all students.

Solution: There are two ways to solve this problem:

Solution 1 (Using the pdf $r(x)$)**:** The median value of x is the number T such that $\int_{-\infty}^{T} r(x)\,dx = 0.5$, or equivalently, $\int_{T}^{\infty} r(x)\,dx = 0.5$. Using the graph of $r(x)$, since

$$\int_{1}^{\infty} r(x)\,dx = \int_{1}^{2} 0.75\,dx = 0.75,$$

we see that we must have $1 < T < 2$. It follows that in the graph of $r(x)$, we are looking for the value of T so that the rectangle with base $[T, 2]$ and height 0.75 has area 0.5. Or, using integrals, we have

$$0.5 = \int_{T}^{\infty} r(x)\,dx = \int_{T}^{2} 0.75\,dx = 0.75(2 - T).$$

We solve the equation $0.5 = 0.75(2 - T)$ for T, which gives $T = 4/3$. Therefore the median value of x is $T = 4/3$.

Solution 2 (Using the cdf $R(x)$)**:** The median value of x is the number T such that $R(T) = 0.5$. Using our graph of $R(x)$ from part (a), we see that $1 < T < 2$. For such T, the slope of $R(x)$ at $x = T$ is $3/4$, so by using the slope formula, we set up an equation involving T and solve:

$$\frac{0.5 - 0.25}{T - 1} = \frac{3}{4} \quad \Rightarrow \quad 4(0.5 - 0.25) = 3(T - 1) \quad \Rightarrow \quad 2 - 1 = 3T - 3 \quad \Rightarrow \quad T = \frac{4}{3}.$$

Therefore the median value of x is $T = 4/3$.

Answer: 4/3