10. [9 points] Let a be a real number. Consider the following integral

$$\int_0^1 ax \ln(x)\,\mathrm{d}x$$

a. [8 points] Show that the above integral converges by using a **direct computation** and find its value in terms of a. Be sure to show your full computation, and be sure to use **proper notation**.

Answer: $\displaystyle\int_0^1 ax \ln(x)\,\mathrm{d}x =$ _____

b. [1 point] Find the value of a so that the function

$$p(x) = \begin{cases} ax\ln(x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function (pdf).

Answer: $a =$ _____