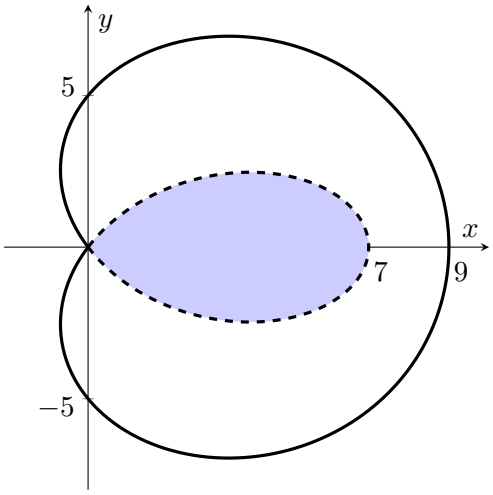
9. [12 points]

Elena, a talented landscape architect, envisions a park whose shape is defined by the polar curve $r = 5 + 8\cos(\theta) - 4\cos^2(\theta)$, as illustrated to the right. In her design, the inner loop of the curve serves as an ideal location for a lake, represented by the shaded region. The solid outer curve in the diagram represents the walking trail that winds around the park.



a. [4 points] Using the factorization $5 + 8\cos(\theta) - 4\cos^2(\theta) = (1 + 2\cos(\theta))(5 - 2\cos(\theta))$, find the values of θ in the interval $[0, 2\pi)$ for which the curve passes through the origin.

Answer: $\theta =$ _____

b. [4 points] To determine the amount of water required to fill the lake, Elena wants to calculate the area of the surface of the lake. Write an expression involving one or more integrals that represents the area of the shaded region. Do not evaluate the integral(s).

Answer: _____

c. [4 points] Recall that the solid outer curve in the diagram represents the walking trail. Write an expression involving one or more integrals that represents the total length of the walking trail. Do not evaluate the integral(s).

Answer: _____