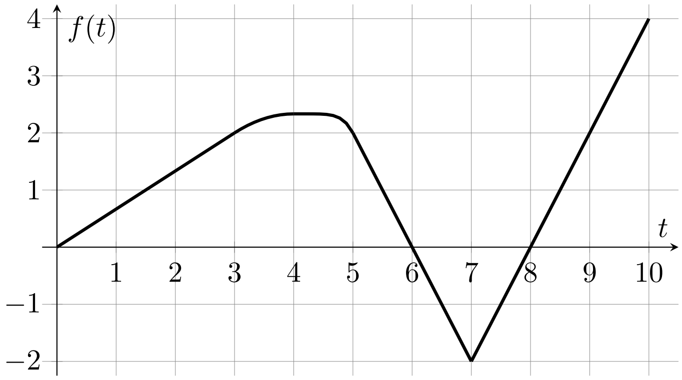
1. [13 points] Caroline uses a remote-controlled boat to survey a reservoir. The boat starts at the point $(x, y) = (0, 0)$, and after t seconds is positioned at $x = f(t)$ and $y = g(t)$. A graph of $f(t)$ and a formula for $g(t)$ are given below. Note that $f(t)$ is linear on the the intervals $[0, 3]$, $[5, 7]$, and $[7, 10]$, and has a local maximum at $t = 4$.



$$g(t) = 12\cos\left(\frac{\pi}{2}t\right) - 12$$

For each of the following parts, your final answer should **not** include the letters f or g.

a. [2 points] Where is the boat located after 10 seconds?

Solution: At $t = 10$, we have $x = f(10) = 4$ and $y = g(10) = 12\cos(5\pi) - 12 = 12(-1) - 12 = -24$.

Answer: $x =$ 4 and $y =$ -24

b. [3 points] Are there any times during these 10 seconds at which the boat comes to a complete stop? If so, list all such times. If not, write NONE.

Solution:
To find when the boat comes to a complete stop, we look for times when both $f'(t) = 0$ and $g'(t) = 0$.
Since $f(t)$ has a local maximum at $t = 4$, it follows that $f'(4) = 0$, and we observe that this is the only value of t for which $f'(t) = 0$. Furthermore, since

$$g'(t) = -6\pi\sin\left(\frac{\pi}{2}t\right),$$

we have $g'(4) = -6\pi\sin(2\pi) = 0$.
Therefore, the boat comes to a complete stop only at $t = 4$.

Answer: $t =$ 4

c. [4 points] Write an expression involving one or more integrals for the total distance traveled by the boat during the **first 3 seconds**. Do not evaluate any integrals in your answer.

Solution: Computing the slope of the line $x = f(t)$ for $0 \le t \le 3$, we note that

$$f'(t) = \frac{2}{3}, \qquad 0 \le t \le 3.$$

Additionally, we have

$$g'(t) = -6\pi \sin\left(\frac{\pi}{2}t\right).$$

Therefore, the total distance traveled by the boat during the first 3 seconds is given by

$$\int_0^3 \sqrt{\left(\frac{2}{3}\right)^2 + \left(-6\pi \sin\left(\frac{\pi}{2}t\right)\right)^2}\, dt.$$

Answer: $\displaystyle\int_0^3 \sqrt{\frac{4}{9} + 36\pi^2 \sin^2\left(\frac{\pi}{2}t\right)}\, \mathbf{d}t.$

d. [4 points] What is the tangent line to the boat's path at $t = 9$? Give your answer in cartesian form.

Solution: Note that

$$f(9) = 2, \qquad \text{and} \qquad g(9) = 12\cos\left(\frac{9\pi}{2}\right) - 12 = -12.$$

Also,

$$\left.\frac{dx}{dt}\right|_{t=9} = f'(9) = \frac{4-(-2)}{10-7} = \frac{6}{3} = 2, \quad \text{and} \quad \left.\frac{dy}{dt}\right|_{t=9} = g'(9) = -6\pi \sin\left(\frac{9\pi}{2}\right) = -6\pi.$$

Therefore, the equation of the tangent line to the boat's path at $t = 9$ is

$$(y - (-12)) = \frac{-6\pi}{2}(x-2) \implies y = -3\pi(x-2) - 12.$$

Answer: $y =$ $-3\pi(x-2) - 12$