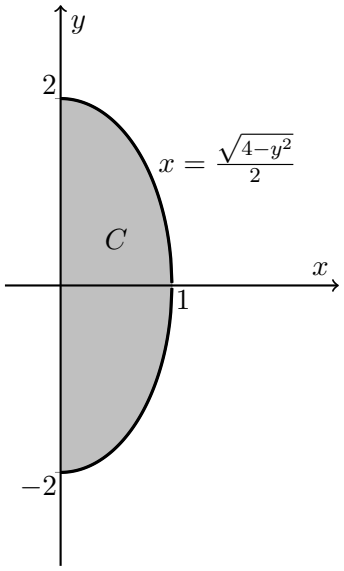
4. [6 points]

Mike owns Mike's Sweet Haven, a bakery that specializes in elegant, custom-made baked goods. With summer approaching, he decides to try something new.

He plans to create a new type of chocolate using the shaded region C, which is bounded by the curves

$$x = \frac{\sqrt{4-y^2}}{2}, \text{ and } x = 0.$$

as illustrated to the right.



a. [4 points] Write an expression involving one or more integrals for the volume of the chocolate obtained by revolving the region C about the y-axis. **Do not** evaluate any integrals in your expression.

Solution: We use horizontal slices to solve this problem. Consider a thin horizontal slice of region C, located y units above the x-axis, with a small thickness Δy. When this slice is rotated about the y-axis, it forms a disk. The inner radius of the disk, denoted by r, is given by

$$r = \frac{\sqrt{4-y^2}}{2}.$$

The approximate volume of this disk is

$$\pi r^2 \Delta y = \frac{\pi(4-y^2)\Delta y}{4}.$$

By integrating from $y = -2$ to $y = 2$, we obtain the total volume of the chocolate:

$$V = \int_{-2}^{2} \frac{\pi(4-y^2)}{4}\, dy.$$

b. [2 points] If Mike were to take a TRAP(4) estimate of the integral you obtained in part **a.**, would he get an underestimate or an overestimate of the volume of the chocolate? No justification is required.

Circle one: **Underestimate** **Overestimate**