8. [6 points] A team of miners is working to extract a box of minerals from a deep pit. The box weighs 40 lbs, and the rope used to lift it weighs 3 lbs per foot. Initially, when the box is at the bottom of the pit, the rope is 60 feet long. As the box is lifted, the miners do not need to lift the portion of the rope that has already been "reeled in", that is, the part that has reached the top of the pit.

a. [3 points] At a certain moment, the box has already been lifted h feet above the ground. Find an expression for the total weight, in pounds, of the box together with the attached rope that has not yet been reeled in.

Solution: We denote by $F(h)$ the total weight, in pounds, of the box together with the portion of rope that has not yet been reeled in at the instant when the box is h feet above the ground. Because the rope was originally 60 feet long, the length still hanging is $60 - h$ feet. Multiplying this length by the rope's weight density and then adding the weight of the box gives

$$F(h) = 3(60 - h) + 40.$$

Answer: $3(60 - h) + 40$

b. [3 points] Using your expression from part (a), find an expression involving one or more integrals that represents the total work done on the box and attached rope, in foot-pounds, to lift the box from the base of the pit to a point 35 feet above its original position. Do not evaluate any integrals that appear in your answer.

Solution: Suppose the box has already been lifted to a height h feet above the ground. The work required to lift it an additional Δh feet is given by:

$$F(h)\,\Delta h = (3(60 - h) + 40)\,\Delta h.$$

Therefore, the total work required to lift the box from the bottom of the pit to a point 35 feet above its initial position is given by

$$W = \int_0^{35} (3(60 - h) + 40)\ \mathrm{d}h.$$

Answer: $\int_0^{35} (3(60 - h) + 40)\ \mathbf{d}h$