11. [9 points] Consider the following initial value problem:

$$y''(t) - 16y(t) = 0$$

with initial conditions $y(0) = 1$ and $y'(0) = 4$. As in the team homework, suppose the solution $y(t)$ to this initial value problem takes the form of a power series centered at $t = 0$:

$$y(t) = \sum_{n=0}^{\infty} c_n t^n = c_0 + c_1 t + c_2 t^2 + \cdots + c_n t^n + \cdots$$

so that we have:

$$y'(t) = c_1 + 2c_2 t + 3c_3 t^2 + \ldots + (n+1)c_{n+1} t^n + \ldots,$$

$$y''(t) = 2c_2 + 3 \cdot 2c_3 t + 4 \cdot 3c_4 t^2 + \ldots + (n+2)(n+1)c_{n+2} t^n + \ldots$$

a. [2 points] Use the initial conditions $y(0) = 1$ and $y'(0) = 4$ to find c_0 and c_1.

Answer: $c_0 =$ ____________________ and $c_1 =$ ____________________

b. [2 points] Substituting $y(t)$ and $y''(t)$ into the differential equation yields the following equation, which must hold for all values of t:

$$\sum_{n=0}^{\infty} \left((n+2)(n+1)c_{n+2} - 16c_n\right) t^n = 0.$$

Use this to find a recurrence relation for c_{n+2} in terms of c_n which applies for all $n \geq 0$.

Answer: $c_{n+2} =$ ____

c. [5 points] We can use the recurrence relation to show that for all $n \geq 0$,

$$c_{2n} = \frac{4^{2n}}{(2n)!} \quad \text{and} \quad c_{2n+1} = \frac{4^{2n+1}}{(2n+1)!}.$$

Use these coefficients to express $y(t)$ as a power series centered around 0 using sigma ($\sum$) notation. Then, use the known Taylor series formula to find a closed-form expression for $y(t)$.

Using sigma notation, $y(t) =$ ____________________________________.

In closed form, $y(t) =$ ____________________________________.