6. [5 points] For the function $f(x) = \cos(\sin(x))$, find $P_2(x)$, the Taylor polynomial of degree 2 for $f(x)$ centered about $x = \pi$. Show all your work.

Solution: To find the Taylor polynomial of degree 2 centered at $x = \pi$, we need to evaluate the function and its first two derivatives at $x = \pi$. First, we compute the derivatives:

$$\begin{aligned} f(x) &= \cos(\sin(x)) \\ f'(x) &= -\sin(\sin(x))\cos(x) \\ f''(x) &= -\cos(\sin(x))\cos^2(x) + \sin(\sin(x))\sin(x) \end{aligned}$$

Now, we evaluate these at $x = \pi$:

$$\begin{aligned} f(\pi) &= \cos(\sin(\pi)) = \cos(0) = 1 \\ f'(\pi) &= -\sin(\sin(\pi))\cos(\pi) = -\sin(0)(-1) = 0 \\ f''(\pi) &= -\cos(\sin(\pi))\cos^2(\pi) + \sin(\sin(\pi))\sin(\pi) = -\cos(0)(-1)^2 + 0 = -1 \end{aligned}$$

Using the formula for the Taylor polynomial $P_2(x) = f(\pi) + f'(\pi)(x-\pi) + \frac{f''(\pi)}{2!}(x-\pi)^2$, we get:

$$P_2(x) = 1 + 0(x-\pi) + \frac{-1}{2!}(x-\pi)^2 = 1 - \frac{1}{2}(x-\pi)^2$$

Answer: $P_2(x) = 1 - \frac{1}{2}(x-\pi)^2$