Math 216 — First Midterm

8 October, 2012

This sample exam is provided to serve as one component of your studying for this exam in this course. **Please note that it is not guaranteed to cover the material that will appear on your exam, nor to be of the same length or difficulty**. In particular, the sections in the text that were covered on this exam may be slightly different from those covered by your exam.

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1. [14 points] Find explicit real-valued general solutions for each of the following. *(Note that minimal partial credit will be given on this problem.)*

a. [7 points] $y' = 2x(e^{-x^2} - y)$

$y =$ _____

b. [7 points] $y'' = -4y' - 13y$

$y =$ _____

2. [14 points] Solve each of the following to find explicit real-valued solutions for y. *(Note that minimal partial credit will be given on this problem.)*

a. [7 points] $y' = x/(y(1+x^2))$, $y(0) = 1$.

$y =$ _____

b. [7 points] $y'' + 14y' + 13y = 0$, $y(0) = 2$, $y'(0) = -2$.

$y =$ _____

3. [8 points] A *Whiffle Ball* is a lightweight plastic ball with holes in at least one hemisphere. If we assume a viscous friction, the upward motion of a thrown or hit whiffle ball may be described in terms of its velocity v or vertical position y by $v' = -\frac{c}{m}\, v - g$ or $y'' = -\frac{c}{m}\, y' - g$. In this problem we take $c/m = 10$ and $g = 10$ (that is, approximately 9.8 m/s^2). If we start with $y(0) = 0$ and $v(0) = 5$ m/s, find the velocity v and position y of the ball.

$v =$ _____

$y =$ _____

4. [15 points] Consider the differential equation $y' = y\,(y^2 + k)$.

a. [4 points] If $k > 0$, find all equilibrium solutions to this equation. Determine the stability of each and draw a phase diagram.

b. [4 points] If $k = 0$, find all equilibrium solutions to this equation. Determine the stability of each and draw a phase diagram.

c. [4 points] If $k < 0$, find all equilibrium solutions to this equation. Determine the stability of each and draw a phase diagram.

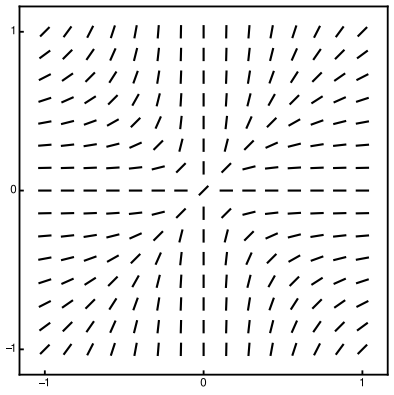
d. [3 points] Use your work from (a)–(c) to draw a bifurcation diagram for this differential equation.

5. [15 points] Let $y_1 = 2\,e^{-x} + 3\,e^{2x}\cos(x)$ and $y_2 = 7\,e^{2x}\sin(x) - 4\,e^{-x}$ be solutions to a homogeneous linear constant-coefficient differential equation.

a. [9 points] Write a possible differential equation of minimal order with these solutions.

b. [6 points] Write the general solution to your differential equation.

6. [8 points] The slope field to the right is that for the differential equation $x^2y' = y^2$, which has solutions $y = x/(Cx+1)$. If we apply the initial condition $y(0) = b$, how does the number of solutions to the initial value problem depend on the value of b? Explain.



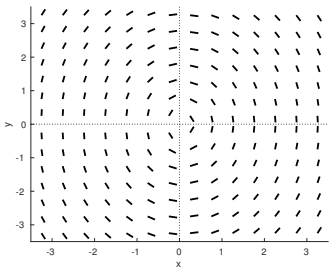
7. [10 points] A very simple model for the deer population P in Michigan is $P' = kP - h$, where k and h are constants, and h is the allowed number of deer that may be killed by hunters each year.

a. [4 points] What is the meaning of the parameter k? Is it positive or negative? Explain.

b. [6 points] Assume that $k > 0$ and solve the differential equation. What does your solution tell you about the long-term deer population?

8. [16 points] Respond to each of the following, giving a *short—one sentence* explanation of your answer. **Note:** *little partial credit will be given on this problem.*

a. [4 points] True or false: the slope field to the right corresponds to the differential equation $y' = x^2 + y^2$. Explain in one sentence.



Answer: _____

b. [4 points] True or false: the function $y = C\,e^{-x}$, where C is an unspecified constant, is the general solution to $y'' + 2\,y' + y = 0$. Explain in one sentence.

Answer: _____

c. [4 points] True or false: if we apply Euler's method and the improved Euler method to $y' = x\,y$, $y(0) = 0$ with step-size $h = 0.1$, both predict after one step that $y(0.1) = 0$. Explain in one sentence.

Answer: _____

d. [4 points] True or false: the graph to the right, below, could be the solution to the differential equation $y' = a^2\,y$ for some value of the constant a.

Answer: _____