**1**. [14 points] Find explicit real-valued general solutions for each of the following. *(Note that minimal partial credit will be given on this problem.)*

**a**. [7 points] $y' = 2x(e^{-x^2} - y)$

$y =$ $\underline{(x^2 + C)\, e^{-x^2}}$

*Solution:* We use the method of integrating factors. Our equation is

$$y' + 2xy = 2xe^{-x^2},$$

so that the integrating factor is $\mu = e^{\int 2x\,dx} = e^{x^2}$. Thus we have

$$(y\,e^{x^2})' = 2x, \qquad \text{so that} \qquad y\,e^{x^2} = x^2 + C.$$

Solving for $y$, $y = x^2e^{-x^2} + Ce^{-x^2}$.

**b**. [7 points] $y'' = -4y' - 13y$

$y =$ $\underline{c_1e^{-2x}\cos(3x) + c_2e^{-2x}\sin(3x)}$

*Solution:* This is a linear, constant coefficient problem, so we guess $y = e^{rx}$. Then $r$ satisfies the characteristic equation $r^2 + 4r + 13 = 0$, or $(r + 2)^2 + 9 = 0$, so $r = -2 \pm 3i$. Thus $y = c_1e^{-2x}\cos(3x) + c_2e^{-2x}\sin(3x)$.