2. [14 points] Solve each of the following to find explicit real-valued solutions for y. *(Note that minimal partial credit will be given on this problem.)*

a. [7 points] $y' = x/(y(1+x^2))$, $y(0) = 1$.

$y =$ $\sqrt{\ln(1+x^2)+1}$

Solution: This is a nonlinear separable problem, $y\,y' = x/(1+x^2)$. Integrating both sides, we have

$$\frac{1}{2}y^2 = \frac{1}{2}\ln(1+x^2) + k, \qquad \text{so that} \qquad y = \pm\sqrt{\ln(1+x^2)+C}.$$

The initial condition $y(0) = 1$ requires that we take the positive sign for the square root and $C = 1$, so we have

$$y = \sqrt{\ln(1+x^2)+1}.$$

b. [7 points] $y'' + 14y' + 13y = 0$, $y(0) = 2$, $y'(0) = -2$.

$y =$ $2e^{-x}$

Solution: This is a linear, constant coefficient problem, so we guess $y = e^{rx}$. Then r satisfies the characteristic equation $r^2 + 14r + 13 = 0$, or $(r+1)(r+13) = 0$, so that $r = -1$ or $r = -13$. Thus the general solution is $y = c_1\,e^{-t} + c_2\,e^{-13x}$. The initial conditions require that $c_1 + c_2 = 2$ and $-c_1 - 13\,c_2 = -2$, so that $c_1 = 2$ and $c_2 = 0$. The solution is thus $y = 2e^{-x}$.