3. [8 points] A *Whiffle Ball* is a lightweight plastic ball with holes in at least one hemisphere. If we assume a viscous friction, the upward motion of a thrown or hit whiffle ball may be described in terms of its velocity v or vertical position y by $v' = -\frac{c}{m}\,v - g$ or $y'' = -\frac{c}{m}\,y' - g$. In this problem we take $c/m = 10$ and $g = 10$ (that is, approximately 9.8 m/s^2). If we start with $y(0) = 0$ and $v(0) = 5$ m/s, find the velocity v and position y of the ball.

$v =$ $6e^{-10t} - 1$

$y =$ $\frac{3}{5}\,(1 - e^{-10t}) - t$

Solution: Solving for v using the method of integrating factors, we have $v' + 10\,v = -10$, or

$$e^{10t}(v' + 10\,v) = (v\,e^{10t})' = -10e^{10t}.$$

Thus, after integrating and applying the initial condition,

$$v\,e^{10t} = -e^{10t} + C = 6 - e^{10t}.$$

We may obtain the same solution by separating variables ($dv/(v{+}1) = 10\,dt$, so that $\ln|v{+}1| = 10t + k$, etc.). We have $v = 6e^{-10t} - 1$. Integrating to find y, we have $y = -\frac{3}{5}\,e^{-10t} - t + C$, so that, for $y(0) = 0$, $y = \frac{3}{5}(1 - e^{-10t}) - t$.