3. [8 points] A Whiffle Ball is a lightweight plastic ball with holes in at least one hemisphere. If we assume a viscous friction, the upward motion of a thrown or hit whiffle ball may be described in terms of its velocity \( v \) or vertical position \( y \) by

\[
\frac{dv}{dt} = -\frac{c}{m} v - g \quad \text{or} \quad \frac{dy}{dt} = -\frac{c}{m} y' - g.
\]

In this problem we take \( c/m = 10 \) and \( g = 10 \) (that is, approximately \( 9.8 \text{ m/s}^2 \)). If we start with \( y(0) = 0 \) and \( v(0) = 5 \text{ m/s} \), find the velocity \( v \) and position \( y \) of the ball.

\[
v = \frac{6e^{-10t} - 1}{6}
\]

\[
y = \frac{3}{5} (1 - e^{-10t}) - t
\]

**Solution:** Solving for \( v \) using the method of integrating factors, we have

\[
e^{10t}(v' + 10v) = (ve^{10t})' = -10e^{10t}.
\]

Thus, after integrating and applying the initial condition,

\[
v e^{10t} = -e^{10t} + C = 6 - e^{10t}.
\]

We may obtain the same solution by separating variables \( (dv/(v+1) = 10 \, dt) \), so that \( \ln |v+1| = 10t + k \), etc.). We have \( v = 6e^{-10t} - 1 \). Integrating to find \( y \), we have \( y = -\frac{3}{5} e^{-10t} - t + C \), so that, for \( y(0) = 0 \), \( y = \frac{3}{5} (1 - e^{-10t}) - t \).