

6. [8 points] The slope field to the right is that for the differential equation  $x^2 y' = y^2$ , which has solutions  $y = x/(Cx+1)$ . If we apply the initial condition  $y(0) = b$ , how does the number of solutions to the initial value problem depend on the value of  $b$ ? Explain.

*Solution:* Looking only at the slope field, we note that the slopes are vertical everywhere along the  $y$ -axis other than at the origin, so we expect no solutions if  $b \neq 0$ . This is supported by the form of the solution, for which  $y(0) = 0$  no matter what  $C$  is. If  $b = 0$ , we see from the slope field and differential equation that  $y = x$  is a solution, and in fact that the given solution works for any value of  $C$ . Thus we expect there to be an infinite number of solutions in this case.

