7. [10 points] A very simple model for the deer population P in Michigan is $P' = kP - h$, where k and h are constants, and h is the allowed number of deer that may be killed by hunters each year.

    a. [4 points] What is the meaning of the parameter k? Is it positive or negative? Explain.

        Solution: The parameter k is the difference between the birth and death rates of the deer. We expect that $k > 0$; if $k < 0$, the death rate exceeds the birth rate and the deer population will go to zero.

    b. [6 points] Assume that $k > 0$ and solve the differential equation. What does your solution tell you about the long-term deer population?

        Solution: We can solve by using an integrating factor or by separating variables. Separating variables, we have $(P - h/k)^{-1} P' = k$, so that $\ln |P - h/k| = kt + \hat{C}$, or $P = C\, e^{kt} + h/k$. Thus if the initial population is larger than h/k we have $C > 0$ and the population will grow to infinity; if the initial population is less than h/k, $C < 0$ and the population will go to zero in finite time.