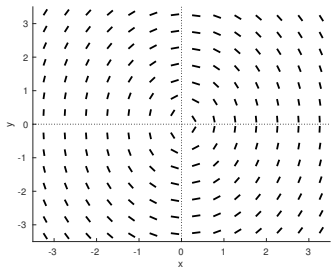
8. [16 points] Respond to each of the following, giving a *short—one sentence* explanation of your answer. **Note:** *little partial credit will be given on this problem.*

a. [4 points] True or false: the slope field to the right corresponds to the differential equation $y' = x^2 + y^2$. Explain in one sentence.



Answer: **False**

Solution: At $(0, 1)$ the slope $y' = x^2 + y^2 = 1$, which is clearly not true for this slope field.

b. [4 points] True or false: the function $y = C\,e^{-x}$, where C is an unspecified constant, is the general solution to $y'' + 2\,y' + y = 0$. Explain in one sentence.

Answer: **False**

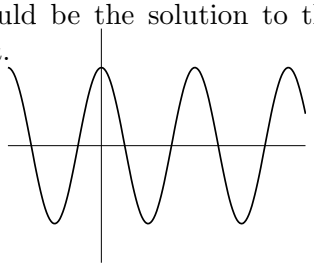
Solution: With $y = e^{rx}$ we get $r^2 + 2r + 1 = (r + 1)^2 = 0$, so the general solution is $y = C_1 e^{-x} + C_2 x e^{-x}$.

c. [4 points] True or false: if we apply Euler's method and the improved Euler method to $y' = x\,y$, $y(0) = 0$ with step-size $h = 0.1$, both predict after one step that $y(0.1) = 0$. Explain in one sentence.

Answer: **True**

Solution: Because at $(0, 1)$ we have the slope $y' = 0$, Euler's method predicts $y(0.1) = 0$; thus both slopes used in in the improved Euler method are zero, and both methods predict $y(0.1) = 0$.

d. [4 points] True or false: the graph to the right, below, could be the solution to the differential equation $y' = a^2\,y$ for some value of the constant a.



Answer: **False**

Solution: All solutions to $y' = a\,y$ are exponential, not sinusoidal.