8. [16 points] Respond to each of the following, giving a short—one sentence explanation of your answer. **Note:** little partial credit will be given on this problem.

a. [4 points] True or false: the slope field to the right corresponds to the differential equation \( y' = x^2 + y^2 \). Explain in one sentence.

**Answer:** False

**Solution:** At \((0, 1)\) the slope \( y' = x^2 + y^2 = 1 \), which is clearly not true for this slope field.

b. [4 points] True or false: the function \( y = Ce^{-x} \), where \( C \) is an unspecified constant, is the general solution to \( y'' + 2y' + y = 0 \). Explain in one sentence.

**Answer:** False

**Solution:** With \( y = e^{rx} \) we get \( r^2 + 2r + 1 = (r + 1)^2 = 0 \), so the general solution is \( y = C_1e^{-x} + C_2xe^{-x} \).

c. [4 points] True or false: if we apply Euler’s method and the improved Euler method to \( y' = xy, \ y(0) = 0 \) with step-size \( h = 0.1 \), both predict after one step that \( y(0.1) = 0 \). Explain in one sentence.

**Answer:** True

**Solution:** Because at \((0, 1)\) we have the slope \( y' = 0 \), Euler’s method predicts \( y(0.1) = 0 \); thus both slopes used in in the improved Euler method are zero, and both methods predict \( y(0.1) = 0 \).

d. [4 points] True or false: the graph to the right, below, could be the solution to the differential equation \( y' = a^2 y \) for some value of the constant \( a \).

**Answer:** False

**Solution:** All solutions to \( y' = ay \) are exponential, not sinusoidal.