4. [18 points] A model for a population with harvesting (e.g., a population of fish from which fish are caught) is \( P' = f(P) = P\left(1 - \frac{P}{K}\right) - H, \) where \( K \) is a limiting population and \( H \) the harvesting rate. \( P \) and \( K \) are measured in some unit—perhaps millions of pounds of fish. Suppose that for some value of \( K \), the graphs of \( f(P) \) are as in the graph shown below.

a. [6 points] Plot phase lines for this equation when \( H = 0, \) \( H = 1 \) and \( H = 2. \) For each, identify all equilibrium solutions and their stability.

b. [5 points] Sketch qualitatively accurate solution curves for the case \( H = 0. \) Include enough initial conditions to show all solution behaviors.
Problem 4, continued. Instructions are reproduced here:

A model for a population with harvesting (e.g., a population of fish from which fish are caught) is 

\[ P' = f(P) = P\left(1 - \frac{P}{K}\right) - H, \]

where \( K \) is a limiting population and \( H \) the harvesting rate. \( P \) and \( K \) are measured in some unit—perhaps millions of pounds of fish. Suppose that for some value of \( K \), the graphs of \( f(P) \) are as in the graph shown below.

\[ f(P) = \begin{cases} 
0 & H = 0 \\
1/2 & H = 1/2 \\
1 & H = 1 \\
3/2 & H = 3/2 \\
2 & H = 2 
\end{cases} \]

\[ P \]

\[ f(P) \]

**c.** [4 points] This problem and your work on it provide an example of a model with a bifurcation. Draw the bifurcation diagram for this on the axes provided below.

\[ P \]

\[ f(P) \]

**d.** [3 points] Explain what your work in the preceding indicates about the long-term survival of the harvested population (fish).