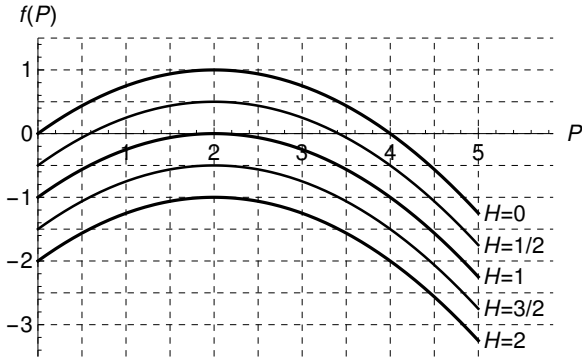
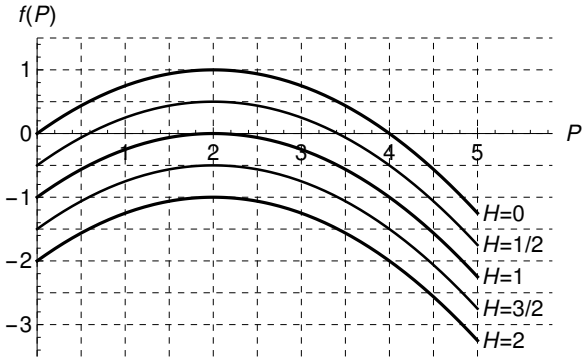
4. [18 points] A model for a population with harvesting (e.g., a population of fish from which fish are caught) is $P' = f(P) = P(1 - \frac{P}{K}) - H$, where K is a limiting population and H the harvesting rate. P and K are measured in some unit—perhaps millions of pounds of fish. Suppose that for some value of K, the graphs of $f(P)$ are as in the graph shown below.

a. [6 points] Plot phase lines for this equation when $H = 0$, $H = 1$ and $H = 2$. For each, identify all equilibrium solutions and their stability.



b. [5 points] Sketch qualitatively accurate solution curves for the case $H = 0$. Include enough initial conditions to show all solution behaviors.

Problem 4, continued. Instructions are reproduced here:
A model for a population with harvesting (e.g., a population of fish from which fish are caught) is $P' = f(P) = P(1 - \frac{P}{K}) - H$, where K is a limiting population and H the harvesting rate. P and K are measured in some unit—perhaps millions of pounds of fish. Suppose that for some value of K, the graphs of $f(P)$ are as in the graph shown below.



c. [4 points] This problem and your work on it provide an example of a model with a bifurcation. Draw the bifurcation diagram for this on the axes provided below.

d. [3 points] Explain what your work in the preceding indicates about the long-term survival of the harvested population (fish).