1. [14 points] Find real-valued solutions for each of the following, as indicated. *(Note that minimal partial credit will be given on this problem.)*

a. [7 points] Find the general solution to $y' = \sin(t) - \frac{\sin(t)}{\cos(t)}\, y$.

Solution: This is linear and not separable, so we must use an integrating factor. The equation may be rewritten as

$$y' + \frac{\sin(t)}{\cos(t)}\, y = \sin(t),$$

so the integrating factor is $\mu = e^{\int \frac{\sin(t)}{\cos(t)}\, dt} = e^{-\ln(|\cos(t)|)} = 1/\cos(t)$ (we may drop the $\pm$ because of the constant of integration which will divide out from the equation after multiplication by μ). Multiplying through by this, we have

$$(\frac{1}{\cos(t)}\, y)' = \frac{\sin(t)}{\cos(t)},$$

so that after integrating both sides we have

$$\frac{1}{\cos(t)}\, y = -\ln(|\cos(t)|) + C,$$

and $y = -\cos(t)\ln(|\cos(t)|) + C\cos(t)$.

b. [7 points] Find a solution, explicit or implicit, for y, if

$$y' = \frac{1 + \sin(t)}{1 + \cos(y)}, \qquad y(\pi) = 0.$$

Solution: This is nonlinear, but separable. Separating variables, we have

$$(1 + \cos(y))y' = 1 + \sin(t).$$

Integrating, we have $y + \sin(y) = t - \cos(t) + C$. Applying the initial condition, we have $0 + 0 = \pi + 1 + C$, so $C = -1 - \pi$, and our (implicit) solution is

$$y + \sin(y) = t - \cos(t) - (1 + \pi).$$