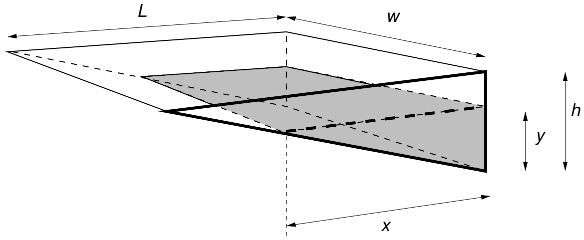
**3**. [12 points] Consider an abandoned zero-entry pool as suggested by the figure to the right, below. (The front face of the pool is shown with bold lines.) In the figure, $w$, $L$ and $h$ are the fixed dimensions of the pool, and $x$ and $y$ characterize the part that is filled with water. The corresponding volume of the filled section is $V = \frac{1}{2}\, w\, x\, y$.




**a**. [8 points] If the pool slowly evaporates at a volumetric rate proportional to its top surface area, write a differential equation for the volume of the water in the pool.

*Solution:* We have $\frac{dV}{dt} = -\alpha\, x\, w$. By similar triangles, $x/y = L/h$, so that $y = hx/L$. Thus we can relate $x$ and $V$ using the volume formula provided: $V = \frac{1}{2}\, w\, x\, y = \frac{hw}{2L}\, x^2$, and so $x = \sqrt{\frac{2L}{hw}}\,\sqrt{V}$. Thus

$$\frac{dV}{dt} = -\alpha\sqrt{\frac{2Lw}{h}}\sqrt{V} = -k\sqrt{V}.$$

**b**. [4 points] Solve your equation from (**a**) with the initial condition $V(0) = V_0$. At what time is the pool finally empty? *(If you are unable to find an equation in (a), you may proceed with the equation $V' = -k\sqrt{V}$.)*

*Solution:* We solve by separating variables: $V^{-1/2}\, V' = -k$, so that on integrating both sides of the equation we have $2\sqrt{V} = -kt + C'$, so that $V = (C - kt/2)^2$. Applying the initial condition, $V(0) = C^2 = V_0$, so that $V = (\sqrt{V_0} - kt/2)^2$. The pool is empty when

$$t = 2\frac{\sqrt{V_0}}{k} = 2\sqrt{\frac{V_0 hw}{L\alpha^2}}.$$