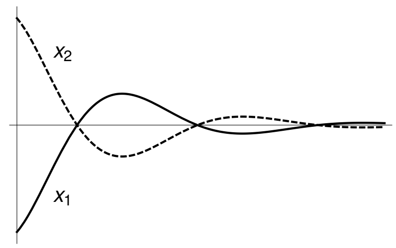
5. [16 points] Consider the system

$$\mathbf{x}' = \mathbf{A}\mathbf{x}, \tag{1}$$

for some real-valued, constant, 2×2 matrix $\mathbf{A}$. Suppose that one solution to (1) is $\mathbf{x} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t}$. Identify each of the following as true or false, by circling "True" or "False" as appropriate, and provide a short (one sentence) explanation indicating why you selected that answer.

a. [4 points] A possible component plot of solutions to (1) is



Solution: This cannot be a correct plot, because the solutions x_1 and x_2 are decaying oscillatory solutions; given one solution with only real exponentials, we know that there can be no oscillatory solutions.

True **[False]**

b. [4 points] The general solution to (1) could be $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{t}$.

[True] False

Solution: This could certainly be correct; we need only that the second eigenvalue of $\mathbf{A}$ be $\lambda = 1$ with corresponding eigenvector $\mathbf{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. (The reversed sign on the first solution is immaterial, as the eigenvectors are only unique up to a constant multiple.)

c. [4 points] The equation $\mathbf{A}\mathbf{w} = -\mathbf{w}$ has infinitely many solutions $\mathbf{w}$.

[True] False

Solution: Because we know one solution to the system is the $\mathbf{x}$ given, we know that one of the eigenvalues of $\mathbf{A}$ is $\lambda = -1$. This is just the eigenvalue problem with $\lambda = -1$ plugged in, so we know that there are an infinite number of solutions.

d. [4 points] An eigenvalue of the matrix $\mathbf{A}$ could be $\lambda = 1 + i$.

True **[False]**

Solution: We know that $\lambda = -1$ is one eigenvalue, that there are at most two (because A is 2×2), and the complex eigenvalues must come as a complex-conjugate pair.