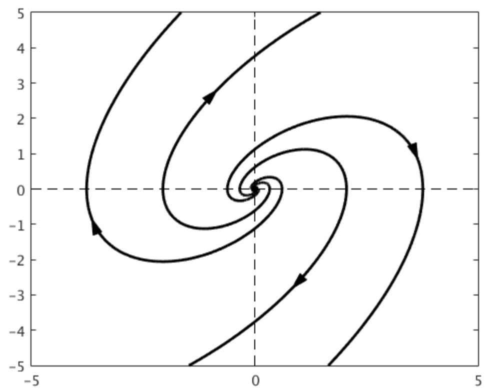
6. [12 points] Consider the system of equations

$$x_1' = x_2$$
$$x_2' = -x_1 + \alpha x_2,$$

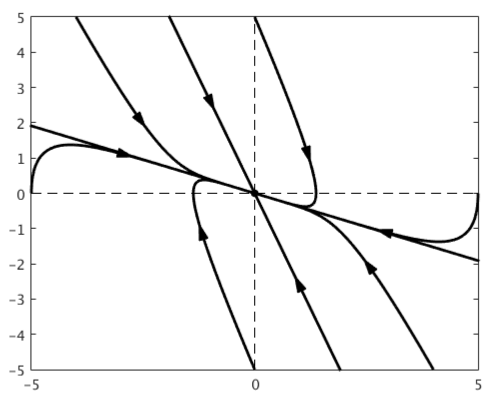
where α is a real-valued constant. For each of the phase portraits shown below, indicate all values for α that could result in this sytem having a phase portrait of that type and with the indicated stability. If it is not possible, write "**not possible**" and give a short explanation why.

a. [6 points]



Solution: For both of these we need the eigenvalues; here, they are given by $\lambda(\alpha-\lambda)+1 = 0$, or $\lambda^2 - \alpha\lambda - 1 = 0$, so $\lambda = \frac{\alpha}{2} \pm \frac{1}{2}\sqrt{\alpha^2 - 4}$. For this phase portrait we need a complex eigenvalue with positive real part, so $0 < \alpha < 2$.

b. [6 points]



Solution: Using the value of λ found above, we require that there be two real, negative roots. Thus we need $\alpha < -2$.