

2. [16 points] Let \mathbf{A} be a 2×2 matrix with real entries that has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 5$ with eigenvectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

a. [6 points] What is the result of each of the following matrix multiplications? Briefly explain your answer for each.

$$\mathbf{A} \begin{pmatrix} -1 \\ 1 \end{pmatrix} =$$

$$\mathbf{A} \begin{pmatrix} 2 \\ 0 \end{pmatrix} =$$

b. [5 points] Sketch a qualitatively accurate phase portrait for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

c. [5 points] Give two initial conditions for which the solution to $\mathbf{x}' = \mathbf{A}\mathbf{x}$ will, as trajectories in the phase plane, eventually be parallel to the line $y = -x$. Give a short explanation of how you know your answer is correct.