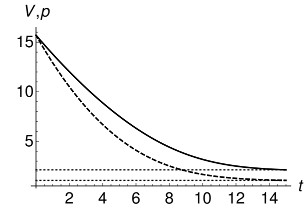
5. [14 points] The volumetric rate at which liquid leaves a cylindrical tank through a circular hole in its bottom is proportional to the square root of the volume of liquid in the tank. Suppose we have a cylindrical tank 5 meters tall with a 1 meter radius (so that its volume is 5π m^3) that is initially full of some liquid. At time $t = 0$, a circular hole opens in the base and more liquid is added at a rate of 1 m^2/hr.

a. [5 points] Write an initial value problem for the volume V of liquid in the tank. (Your answer will involve a constant of proportionality k.) Can you solve your equation? Explain. *(Do not actually solve the equation.)*

b. [5 points] Suppose that the solution to your equation in (a) is some function $V(t)$. If the liquid in the tank initially contains a particulate at a concentration of 1 g/m^3 and the liquid entering has a particulate concentration of 2 g/m^3, write an initial value problem for the amount of particulate in the tank. (Your answer will involve the unknown function $V(t)$.) Can you solve this equation? Explain. *(Do not actually solve the equation.)*

Problem 5, continued.

c. [4 points] What do you expect the long-term value for the volume $V(t)$ to be? Can you predict the long-term value for $p(t)$? If $k = 1$, which of the graphed functions to the right is $V(t)$ and which is $p(t)$? Why?



6. [10 points] Consider the initial value problem $(1 - y^3)\frac{dy}{dt} = 1$, $y(0) = 0$.

a. [5 points] Without solving it, will this initial value problem have a unique solution?

b. [5 points] Solve the problem. Based on your solution, for what range of t and y values would you expect the solution to exist? Why?