

2. [15 points] Consider the direction field shown to the right, which corresponds to a first order differential equation $y' = f(t, y)$.

a. [5 points] Which of the following functions $f(t, y)$ is most likely to be the function in this differential equation? Briefly explain how you made your choice.

$$f(t, y) = (y + 1)(y - 1)$$

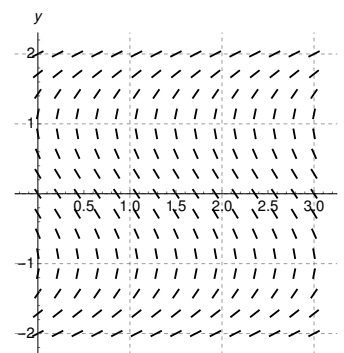
$$f(t, y) = \frac{2}{(y+1)(y-1)}$$

$$f(t, y) = \frac{\sin(\frac{\pi}{2}t)}{y^2-1}$$

$$f(t, y) = \sin(\frac{\pi}{2}y)$$

$$f(t, y) = \frac{2}{\sin(\frac{\pi}{2}y)}$$

$$f(t, y) = \frac{y+1}{y-1}$$



b. [5 points] Sketch, on the direction field or below, the solution to $y' = f(t, y)$, $y(1) = 0$. For what values of t and y will it exist (you should be able to determine these without calculations)? Why?

c. [5 points] Based on your choice of $f(t, y)$ in (a) and the corresponding direction field, are there any initial conditions (t_0, y_0) for which you cannot guarantee that there exists a unique solution? Explain.