4. [12 points] For each of the following give an example, as indicated. It may be useful to note that the eigenvalues and eigenvectors of the matrix $\mathbf{A} = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix}$ are $\lambda = 1$, $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda = 2$, $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

a. [3 points] Give an example of a linear first-order equation that is not separable.

b. [3 points] Give two distinct, non-zero solutions $\mathbf{x}_1$ and $\mathbf{x}_2$ to the system $\mathbf{x}' = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix} \mathbf{x}$ for which $\mathbf{x} = c_1\mathbf{x}_1 + c_2\mathbf{x}_2$ is not a general solution to the system.

c. [3 points] Give a 2×2 matrix $\mathbf{B}$, with all non-zero entries, for which $\mathbf{Bx} = \mathbf{0}$ has an infinite number of solutions.

d. [3 points] Give three different vectors, $\mathbf{w}_1$, $\mathbf{w}_2$, and $\mathbf{w}_3$, for which $\begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix} \mathbf{w}_j = k\,\mathbf{w}_j$, for some k. (The value of k need not be the same for all three vectors.)