5. [16 points] Recall that the van der Pol equation we studied in lab 1 is given by

$$x'' + \mu f'(x)x' + x = 0,$$

or, as a system in x and $y = x'$,

$$x' = y, \qquad y' = -x - \mu f'(x)y,$$

for some function $f'(x)$. We assume that $\mu > 0$.

a. [3 points] Show that for any choice of $f'(x)$, the only critical point of the system formulation of the van der Pol equation is $(0, 0)$.

b. [4 points] Suppose that the Taylor expansion of $f'(x)$ around $x = 0$ is $f'(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + \cdots$. Use this expansion to linearize your system. Your linear system will involve the coefficients a_n.

Problem 5, continued. We are considering the van der Pol system.

c. [5 points] Suppose that the system you obtained in (b) is $x' = -a_0\mu x - y$, $y' = x$. For what value or values of a_0 will the phase portrait for this system have only one straight-line of solution trajectories? No straight-line trajectories? Explain.

d. [4 points] When a_0 is picked so that there is a single straight-line of solution trajectories in the phase portrait for this system, give an initial condition that will result in a straight-line trajectory in the phase plane.